

# AN INTERPRETATION OF COSMOLOGICAL MODEL WITH VARIABLE LIGHT VELOCITY

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**ABSTRACT :** A cosmological model with variable  $c$ ,  $h$ ,  $G$  is proposed. The characteristic lengths of physics (Compton, Jeans, Schwarzschild) are assumed to vary like  $R(t)$ . Both light and matter's worlds are found to obey the same law  $R \approx t^{2/3}$ . The Planck constant is found to vary like  $t$  and the gravitation one like  $1/R$ , while the Planck length vary like  $R$ . The particle masses follow  $m \cong R$ . The Hubble law still apply. The redshifts come from the secular variation of the Planck constant.

## 1 - INTRODUCTION

Since 1930 the constancy of several so called constants of physics have been criticized by many authors [1,2,3,4]. Accurate laboratory measurements show that these values appear quite constant in our today's spacetime field, which is very small with respect to the whole spacetime, although Van Flandern [6] claimed for observational evidence of the variation of the gravity constant  $G$ . As far as we can see the extension of the constancy of the light velocity, and others so called "fundamental constants" over the overall cosmic scale is a still debatable hypothesis. The purpose of this paper is to examine some of the consequences of a model in which the "constants" (especially the light velocity) are assumed to vary with time.

## 2) THE POSSIBLE SECULAR VARIATION OF $c$

Milne [1] tried the first to propose an attempt of this type. He suggested that the observed red shifts are due to some secular change of the Planck constant and not to the classical Doppler effect. If the energy of the traveling photon remains constant, the apparent decrease of the observed frequency would be only due to the linear increase of  $h$  with the cosmic time  $t$ . In addition Milne [1] suggested a decrease in time of the gravity constant  $G$ .

Similarly Fred Hoyle [2] argued against the assumption of the constancy of the mass content of the universe. He also suggested a secular change of  $G$  and constant creation of matter. Dirac [3,4], starting from an hypothesis about the variation in time of some large numbers, built with characteristic physical quantities (like the ratio of the electromagnetic force to the gravitational force), got a variable  $G$  and a constant creation of matter. Later Canuto and Hsieh [8], Lodenquai [5] and Julg [7] explored some consequences of Dirac's initial idea. But, surprizingly, no one contested the absolute constancy of  $c$ .

In the field equation the so called Einstein constant  $\chi$  is determined by identification to the Poisson equation, which gives:

(1)

$$\chi = - \frac{8 \pi G}{c^2}$$

The quantity  $\chi$  must be an absolute constant with respect to the four dimensions, for the field equation to be divergenceless. But once the identification mentioned above refers to a steady situation it does not imply the absolute constancy of  $G$  and  $c$ . A world model could *a priori* be built, with variable  $G$  and  $c$ , with respect to the cosmic time (which will be defined later), if the ratio  $G/c^2$  remains an absolute constant.

In the sequel of the paper we are going to analyze the effects of a secular variation of the light velocity.

### 3. SUGGESTING GAUGE RELATIONS

The Robertson-Walker metric, based on the isotropy and homogeneity assumptions, leads to the following system:

(2)

$$- \frac{8 \pi G}{c^2} \rho = \frac{3k}{R^2} + \frac{3}{R^2} \left( \frac{dR}{dx^0} \right)^2$$

(3)

$$\frac{8 \pi G}{c^2} \left( \frac{p}{c} \right) = - \frac{k}{R^2} - \frac{1}{R^2} \left( \frac{dR}{dx^0} \right)^2 - \frac{2}{R} \frac{d^2 R}{dx^{0^2}}$$

In this system,  $k$  is the sign of the curvature,  $p$  the pressure and  $\rho$  the density of energy-matter. In the classical model we define the cosmic time  $t$  from the chronological variable  $x^0$ , by  $x^0 = ct$ , where  $c$  is considered as an absolute constant. In addition the wavelength of the photon varies like  $R$ .

Let us consider the less restrictive following condition:

(4)

$$d x^\circ = c(t) dt$$

which represents an alternative interpretation of the chronological parameter  $x^\circ$ . We are now going to relate the main physical constants to  $R$ , considered as a gauge parameter:

(5)

$$c \approx \frac{1}{\sqrt{R}} \quad \text{or} \quad R c^2 \approx \text{constant}$$

(5)

$$m \text{ ( particle's mass ) } \approx R$$

(6)

$$h \approx R^{3/2}$$

(7)

$$G \approx 1/R$$

Referring to the relation (1) notice that  $G/c^2 = \text{constant}$ . In addition if  $V$  is the relative velocity of a given element, for example the random velocity of a galaxy in a cluster, or the velocity of a free particle in a cloud, we assume that  $V$  follows the secular variation:

(9)

$$V \approx R^{-1/2}$$

If we suppose that the number of particles is conserved, the matter density  $\rho$  obeys:

(10)

$$\rho \approx 1/R^2$$

As a consequence we can express the cosmic evolution through a gauge process, i.e. the Compton wavelength, the De Broglie wavelength, the Schwarzschild length and the Jeans length are found to vary like  $R$ .

In addition our model still considers  $mc^2 = \text{constant}$  and:

(11)

$$\frac{m c^2}{\sqrt{1 - \frac{V^2}{c^2}}} \approx \text{constant}$$

The classical model saved the masses, as assumed to be constant, but not the total matter-energy, through the variation of the the cosmic background energy. In our scenario it is just the reverse: the

energy-matter is constant in time, not the masses. In addition, one should notice that the quantity  $Gm^2/R$ , which can be considered as a characteristic gravitational energy, is conserved.

Since the energies are conserved in our model, the moment as defined as  $mVi$  varies like  $R^{1/2}$ . It is only constant if we define them as  $\rho \dot{V} c$ .

Finally the Planck length varies with time like  $R(t)$ , the Planck time varies like  $t$  and the gravitational forces like  $1/R(t)$ .

#### 4. THE EVOLUTION EQUATION

Introducing (4) into the system (2),(3) we obtain the following equations :

(12)

$$\frac{2R''}{R} + \frac{2R'^2}{R^2} + \frac{kc^2}{R^2} = \alpha \rho$$

(13)

$$\frac{1}{R^2} (R'^2 + kc^2) = -\alpha \frac{\rho c^2}{3}$$

The use of the following equation of state

(14)

$$\rho = \frac{\beta^2 c^2}{3} \quad \text{with} \quad 0 < \beta \leq 1$$

leads to:

(15)

$$\frac{2R''}{R} + \frac{R'^2}{R^2} (2 + \beta^2) + \frac{kc^2}{R^2} (1 + \beta^2) = 0$$

In the case where  $R = a t^m$  the parameter  $\beta$  disappears from (15). From (5)  $Rc^2 = R_0 c_0^2$  is an absolute constant,  $R_0$  and  $c_0$  being the present values of the gauge parameter  $R$  and light velocity  $c$ . The only possible value for  $k$  is  $-1$ , which means that in our model the curvature is negative. Then the evolution law becomes:

(16)

$$R = \sqrt[3]{\frac{9}{4} R_0 c_0^2} t^{\frac{2}{3}}$$

Here, at contrary to the classical models, light and matter have the same evolution law. Moreover:  
(17)

$$R = \frac{3}{2} c t$$

If we know to, the age of the universe, and  $c_0$ , the present value for the light velocity, we could derive the present value of the gauge parameter of the universe  $R_0 = (3/2) c_0 t_0$ , with the use of:  
(18)

$$L(t) \equiv \int_0^t c(\tau) d\tau \equiv R(t)$$

The consequence is that the horizon is found to be identical, at any time, to the gauge factor  $R(t)$ .

## 5. GAUGE INVARIANCE OF SOME FUNDAMENTAL EQUATIONS

Let us take first the Vlasov equation, referring to collision free fluids.  $f(\mathbf{r}, \mathbf{V}, t)$  is the velocity distribution function, which depends on the position vector  $\mathbf{r}$ , the velocity vector  $\mathbf{V}$  and the time  $t$ .  $\Psi$  is the gravitational, so that  $-m \partial \Psi / \partial \mathbf{r}$  is the force acting on the particle whose mass is  $m$ .  
(19)

$$\frac{\partial f}{\partial t} + \mathbf{V} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Psi}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{V}} = 0$$

Introduce non-dimensional variables, such that:

$$t = t^* t ; f = f^* x ; \mathbf{V} = V^* \mathbf{w} ; \mathbf{r} = R^* \mathbf{z} ; \Psi = (G^* m^* / R^*) \varphi$$

The equation (19) becomes:  
(20)

$$\frac{1}{t^*} \frac{\partial x}{\partial \tau} + \frac{V^*}{R^*} \mathbf{w} \cdot \frac{\partial x}{\partial \xi} - \frac{G^* m^*}{R^{*2} V^*} \frac{\partial \varphi}{\partial \xi} \cdot \frac{\partial x}{\partial \mathbf{w}} = 0$$

Introduce the precedent gauge relations  $G^* \approx 1/R^*$ ,  $m^* \approx R^*$ .

The dimensional analysis of the equation (20) gives  $V^* \approx 1/(R^*)^{1/2}$  and:  
(21)

$$R^* \approx t^{*2/3}$$

These relations can be interpreted as gauge relations and related to the solution (16). Consider now the Schrödinger equation:  
(22)

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + U \Psi = -i \hbar \frac{\partial \Psi}{\partial t}$$

Introduce :

$$t = t^* t, \mathbf{r} = R^* \mathbf{z}, h = h^* h, m = m^* m, U = U^* u.$$

The dimensional analysis of the equation (22) gives :

(23)

$$\frac{h^{*2}}{2 m^* R^{*2}} \approx U^* \approx \frac{h^*}{t^*} \quad \text{i.e.} \quad \frac{t^{*2}}{R^*} \approx U^* \approx \text{constant}$$

i.e.  $R^* \approx t^{*2/3}$ . Now let us write the Maxwell equations, referring to an empty space:

(24)

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

(25)

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$$

and write:

$$\mathbf{E} = E^* \boldsymbol{\varepsilon}, \mathbf{B} = B^* \boldsymbol{\beta}, \mathbf{r} = R^* \boldsymbol{\zeta}, t = t^* t, c = c^* \omega$$

We get:

(26)

$$\frac{E^*}{R^*} \frac{\partial}{\partial \boldsymbol{\zeta}} \times \boldsymbol{\varepsilon} = - \frac{B^*}{t^*} \frac{\partial \boldsymbol{\beta}}{\partial \tau}$$

(27)

$$\frac{E^*}{R^*} \frac{\partial}{\partial \boldsymbol{\zeta}} \times \boldsymbol{\beta} = \frac{E^*}{c^{*2} t^*} \frac{\partial \boldsymbol{\varepsilon}}{\partial \tau}$$

Combining with  $c^* \approx 1/R^{*1/2}$  we find  $R^* \approx t^{*2/3}$ .

## 6. CONCLUSION.

In this paper we have derived some of the implications of letting the fundamental constants vary with time. This can only be done with the addition of some further gauge constraints. Following the Milne's suggestion [1] the classical interpretation of the red shift in terms of Doppler effect has to be replaced by another one taking account the secular change of the Planck constant. The fundamental parameters  $R$  and  $c$  are related to each other by some gauge relation. The particle masses vary like  $R$  while the energy-matter and the gravitation energy are conserved.

This model predicts that the cosmological horizon  $L(t)$  should be identical to  $R(t)$ , which would justify the overall homogeneity of universe. The curvature of space should be negative and the gauge relationship between  $R$  and  $t$  should be  $R \sim t^{2/3}$ .

The Planck's constant would vary like  $t$ , and the gravitational constant  $G$  like  $1/R$ , such as the Planck's length would vary like  $R$ , as well as the Planck's time would vary like  $t$ . The gravitational force would vary like  $1/R$ .

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