

## Negative mass hypothesis in cosmology and the nature of dark energy

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The observed acceleration of the universe arises a puzzling question. What is the nature of a dark energy that would cause this phenomenon ? We recall the arguments against the existence of negative matter on General Relativity grounds. They vanish if the universe is considered as a manifold  $M_4$  associated to two coupled metrics, solutions of a coupled field equation system. We build a non-steady solution where the positive species accelerates while the negative one decelerates. So that dark energy is replaced by (dominant) negative matter action.

### I. Introduction

As pointed out by Riess & al. 1998 [1]; Perlmutter & al. 1999 [2], reviewed by Riess 2000 [3]; Filippenko 2001[4]; Leibundgut 2001[5], observations of type Ia supernovae at redshift  $z < 1$  rise a serious problem, showing that the universe accelerates, this phenomenon being attributed to some puzzling « dark energy ». Expanding this investigation on SNe Ia comforts such conclusion (Knop et al.2003 [6]). This has been extended to larger redshifts samples (Tonry et al. 2003 [7]; Barris et al. 2004 [8]). See also Riess and al. 2004 [9]. Nobody anticipated such startling phenomenon. The magnitude of the observed acceleration was not anticipated by theory and continues to defy a post facto explanation. What this newly discovered component of the universe could be made of ?

### II. Can negative mass play a role in physics ?

The observed acceleration of the universe suggests the effect of a negative pressure, associated to a negative mass content. Is it possible that the Universe would contain negative energy particles ? The dynamic group theory [10] shows that such concept

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comes from the antichron components of the Poincaré group. This last contains four connex components. The first four form the ~~antichron~~ orthochron subgroup, or restricted Poincaré group. The last four form the antichron subset, whose elements reverse time. The coadjoint action of this group on its momentum shows that time inversion goes with energy, and mass inversion. Quantum theory of fields analyzes the inversion of space and time too. In dynamical groups theory groups are built with real components. In QFT the T operator become complex, so that it can be either linear and unitary, either antilinear and antiunitary. In [11], page 75-76 Weinberg writes, we quote :

“At this point we have not *decided* whether P and T are linear and unitary or antilinear and antiunitary. The *decision* is an easy one. Setting  $\rho = 0$  in Eq.(2.6.4) gives  $P i H P^{-1} = - i H$ , where  $H \equiv P^0$  is the energy operator . If P were antiunitary and antilinear then it would anticommute with  $i$  , so  $P H P^{-1} = - H$  . But for any state  $\Psi$  of energy  $E > 0$  , there would be another state  $P^{-1} \Psi$  of energy  $-E$  . *There are no states of negative energy* (energy less than that of the vacuum) , so we are *forced to choose* the other alternative : P is *linear and unitary* and commutes than anticommutes with  $H$ . On the other hand, setting  $\rho = 0$  in Eq.(2.6.6) yields  $T i H T^{-1} = - i H$ . If we supposed that T is linear and unitary we could simply cancel the  $i$ s, and find  $T H T^{-1} = - H$ , with the again *disastrous conclusion* that for any state  $\Psi$  of energy  $E$  there is another state  $T^{-1} \Psi$  of energy  $- E$ . *To avoid this, we are forced to conclude* that T is antilinear and antiunitary”. We see than the quantum theory of fields is built from *a priori choices about fundamental operators*, which are done in order to *avoid the disastrous fact to include negative energy particles in the theory*”.

This absence is nothing but a pure hypothesis. A stronger critic comes from General Relativity theory and was pointed out in 1957 by H.Bondi [12]. Newton’s law emerges naturally from Einstein’s equation, through Newtonian approximation. Considering the action of a particle (2) on a particle (1), Bondi defines three kinds of masses :

$$m_i^{(1)} \frac{d^2 \vec{r}_1}{dt^2} = \frac{G (r_2 - r_1) m_p^{(1)} m_a^{(2)}}{|\vec{r}_2 - \vec{r}_1|^3} \quad (1)$$

$m_i$  is the inertial mass,  $m_p$  the passive gravitational mass and  $m_a$  the active gravitational mass. Immediatly the equivalence principle gives  $m_i = m_p$  so that (1) becomes :

$$\frac{d^2 \vec{r}_1}{dt^2} = \frac{G (r_2 - r_1) m_a^{(2)}}{|\vec{r}_2 - \vec{r}_1|^3} \quad (2)$$

which shows that positive mass attracts both positive and negative matter. Negative mass repels the two. So that two negative masses repel each other and cannot form clumps by gravitational instability. On another hand, if two masses with same

absolute values and opposite signs encounter, the positive one escapes, while the negative one runs after it. The two experience a constant acceleration, but energy and momentum are conserved, because of the two masses are negative. Such preposterous features banned negative mass from cosmology during more than half a century.

### III. "Regions that behave as if they were filled by negative matter"

Several authors suggested that the voids are a fundamental element of the large scale structure (LSS), with bubble-like structure ([13] to [21]). In 1997 T. Piran [22] suggested that initial tiny perturbations, in the primordial density field, "acting as if the had a negative mass", could extend and play a role in the LSS. He made the hypothesis, without justification, that negative mass could own a negative active mass (from Bondi's definition), but positive inertial and passive masses, so that two negative masses would mutually attract. Negative material could form clusters. Positive and negative masses would repel each other, which was studied in references [17] and [18]. Piran developed a Void Finder Algorithm which identifies voids in reshift surveys and measure their size and underdensity. In a more recent paper (2013) Koji Izumi, Chizaki Hagiwara, Koki Nakajima, Takao Kitamura and Hideki Asada [23] consider gravitational lensing shear by the magnifying lens models such as negative mass compact objects causing gravitational repulsion on light, like a concave lens. They suggest, as on reference [6] that negative masses might attract each other, and form negative massive clumps, so that they could reside in cosmological voids (and there, they mention Piran's work). The demagnifying effect was previously predicted in [6], due to *negative lensing effect*. In addition they have shown that the images by the lens models are radially distorted, due to the repulsive effect (instead tangentially, when due to positive masses).

Classically, the observed distortion of the images of galaxies (weak lensing effect) is used to determine the three-dimensional distribution of some hypothetical CDM. Conversely, when decoding the data, if one assumes that the observed distortion is radial, instead tangential, it could provide three-dimensional negative matter distribution. If this last is found as a regular set of clumps, located at the center of the observed big voids that would be in favour of the presence of such negative mass clumps in our universe.

### IV. Towards a bimetric description of the Universe

General relativity is based on a  $M_4$  manifold, associated to a single Riemannian metric, considered as solution of Einstein's field equation. On such grounds, negative matter cannot cohabit with positive matter. What about a different geometrical

framework ? A first bimetric description was proposed in [17], [18] and [24], then in [25] and [26]. Let us consider the following coupled field equations system :

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi (T_{\mu\nu}^{(+)} + \phi T_{\mu\nu}^{(-)}) \quad (3a)$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi (\phi T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}) \quad (3b)$$

$g_{\mu\nu}^{(+)}$  metric refers to positive energy (and mass if they have one) particles.  $g_{\mu\nu}^{(-)}$  refers to *negative* energy (and mass if they have one) particles.  $R_{\mu\nu}^{(+)}$  and  $R_{\mu\nu}^{(-)}$  are Ricci tensors, built from metrics  $g_{\mu\nu}^{(+)}$  and  $g_{\mu\nu}^{(-)}$ . For sake of brevity we will sometimes use the notation :  $f \in \{+, -\}$ . In mixed form, we will write the tensors :

$$T_{\nu}^{(f)\mu} = \begin{pmatrix} \rho^{(f)} c^2 & 0 & 0 & 0 \\ 0 & -p^{(f)} & 0 & 0 \\ 0 & 0 & -p^{(f)} & 0 \\ 0 & 0 & 0 & -p^{(f)} \end{pmatrix} \quad \text{with} \quad \begin{array}{ll} \rho^{(f)} > 0 & \rho^{(f)} < 0 \\ p^{(f)} > 0 & p^{(f)} < 0 \\ \text{For } f = "+" & \text{For } f = "-" \end{array} \quad (4)$$

$\phi$  and  $\phi$  are functions that will be defined further (to ensure energy conservation). Each metric produces its own geodesics family. The two are distinct, so that opposite energy particles cannot encounter, on pure geometric grounds. They interact only through gravitation. We assume that positive light, composed by positive energy photons, cruises along null geodesics of metric  $g_{\mu\nu}^{(+)}$ , while negative light, composed by negative energy photons, cruises along null geodesics of metric  $g_{\mu\nu}^{(-)}$ . We assume that structures made of positive masses can emit and capture positive energy photons. Similarly, structures made of negative masses can emit and capture negative energy photons. So that on geometric grounds, negative structures would be invisible to us, through our eyes or astronomic instruments.

## V. Non-steady solution for dust universe

We assume this bimetric universe is homogeneous and isotropic, so that the Riemannian metrics become :

$$(ds^{(f)})^2 = c^2 dt^2 - (a^{(f)})^2 \frac{du^2 + u^2(d\theta^2 + \sin^2 \theta d\varphi^2)}{(1 + k^{(f)} \frac{u^2}{4})^2} \quad (5)$$

Introducing these metrics in the system (3a) + (3b), with  $p^{(+)} \cong 0$  and  $p^{(-)} \cong 0$  we get classical systems :

$$\frac{3}{c^2 (a^{(+)} )^2} \left( \frac{da^{(+)}}{dt} \right)^2 + \frac{3k^{(+)}}{c^2 (a^{(+)} )^2} = -\chi c^2 [\rho^{(+)} + \varphi \rho^{(-)}] \quad (6a)$$

$$\frac{2}{c^2 (a^{(+)} )^2} \frac{d^2 a^{(+)}}{dt^2} + \frac{1}{c^2 (a^{(+)} )^2} \left( \frac{d a^{(+)}}{dt} \right)^2 + \frac{k^{(+)}}{c^2 (a^{(+)} )^2} = 0 \quad (7a)$$

$$\frac{3}{c^2 (a^{(-)} )^2} \left( \frac{da^{(-)}}{dt} \right)^2 + \frac{3k^{(-)}}{c^2 (a^{(-)} )^2} = +\chi c^2 [\phi \rho^{(+)} + \rho^{(-)}] \quad (6b)$$

$$\frac{2}{c^2 (a^{(-)} )^2} \frac{d^2 a^{(-)}}{dt^2} + \frac{1}{c^2 (a^{(-)} )^2} \left( \frac{d a^{(-)}}{dt} \right)^2 + \frac{k^{(-)}}{c^2 (a^{(-)} )^2} = 0 \quad (7b)$$

Applying classical mathematical methods [27] the compatibility conditions of sets (6a+7a) and (6b+7b) gives :

$$3 \frac{da^{(+)}}{a^{(+)}} + \frac{d[\rho^{(+)} + \varphi \rho^{(-)}]}{[\rho^{(+)} + \varphi \rho^{(-)}]} = 0 \quad (8a)$$

$$3 \frac{da^{(-)}}{a^{(-)}} + \frac{d[\phi \rho^{(+)} + \rho^{(-)}]}{[\phi \rho^{(+)} + \rho^{(-)}]} = 0 \quad (8b)$$

The conservation of the energy ( and mass ) :

$$E = \rho^{(+)} c^2 (a^{(+)})^3 + \rho^{(-)} c^2 (a^{(-)})^3 \quad (9)$$

is ensured if :

$$\varphi = \left( \frac{a^{(-)}}{a^{(+)}} \right)^3 \quad \phi = \left( \frac{a^{(+)}}{a^{(-)}} \right)^3 \quad \phi = \varphi^{-1} \quad (10)$$

So that our coupled field equation system becomes :

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left[ T_{\mu\nu}^{(+)} + \left( \frac{a^{(-)}}{a^{(+)}} \right)^3 T_{\mu\nu}^{(-)} \right] \quad (11a)$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[ \left( \frac{a^{(+)}}{a^{(-)}} \right)^3 T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)} \right] \quad (11b)$$

Let :

$$\chi = -\frac{8 \pi G}{c^4} \quad (12)$$

$$(a^{(+)})^2 \frac{d^2 a^{(+)}}{dt^2} = -\frac{4 \pi G}{c^4} E \quad (13a)$$

$$(a^{(-)})^2 \frac{d^2 a^{(-)}}{dt^2} = +\frac{4 \pi G}{c^4} E \quad (13b)$$

## VI. The nature of "dark energy"

Assume  $E < 0$ . Then  $a^{(+)} > 0$  and  $a^{(-)} < 0$ . If we suppose that our visible part of the universe corresponds to positive mass, then it accelerates, while the negative species decelerates. Equation (13b) identifies to Friedman equation, while (13a) identifies to Bonnor's solution [28] :

$$\begin{aligned} a^{(+)}(u) &= \alpha^2 c h^2 u \\ t^{(+)}(u) &= \alpha^2 \left( 1 + \frac{sh 2u}{2} + u \right) \end{aligned} \quad (14)$$

From dynamic group theory [10] all particles, including photons, may have negative mass. If  $L_o$  is the element of the orthochron restricted Lorentz group, let us consider the following extension of the Poincaré group.

$$\begin{pmatrix} \lambda \mu & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with } \lambda = \pm 1 \quad \text{and} \quad \mu = \pm 1 \quad (15)$$

which acts on a five-dimensional Kaluza space. The dimension of the group is 11. Then the momentum gets an extra scalar  $q$ , identified by J.M.Souriau as the electric charge [29]. The coadjoint action of the group gets an additional relation :

$$q' = \lambda \mu q \quad (16)$$

Matter-antimatter duality holds in the negative mass world. The negative matter, which acts as "dark energy" is composed by negative energy (and negative mass, if they have one) particles : electrons, protons, neutrons, neutrinos, quarks, an so on, plus their antiparticles, plus photons.

## VII. Newtonian approximation.

To develop approximation approximation we need to expand the metrics into series around Lorentzian metrics. The Universe is not homogeneous. When we develop classical Newtonian approximation we assume that the zeroth order term is Lorentzian, which means that this metric is solution of the field equation with a null second member, describing an empty portion of space. It is not necessary that this zeroth order solution corresponds to a description of the whole universe, only a restricted portion of space, small enough to encompass the interaction of particles, planets or stars, considered as mass-points. Similarly, whatever is the time-dependent model, it is possible to consider that the system is quasi steady. In our bimetric description, we consider a finite portion of quasi-steady empty space-time (null curvature, Lorentzian metrics).

$$g^{(+)} = \eta^{(+)} + \varepsilon \eta^{(+)} \quad g^{(-)} = \eta^{(-)} + \varepsilon \eta^{(-)} \quad (17)$$

The two scale factor  $a^{(+)}$  and  $a^{(-)}$  are considered as constants. Then, field equations are expanded into a series. Neglecting the second order terms, we find :

$$\varepsilon \gamma_{00|\beta|\beta}^{(+)} = -\chi c^2 \left[ \delta \rho^{(+)} + \left( \frac{a^{(-)}}{a^{(+)}} \right)^3 \delta \rho^{(-)} \right] \quad (18a)$$

$$\varepsilon \gamma_{00|\beta|\beta}^{(-)} = \chi c^2 \left[ \left( \frac{a^{(+)}}{a^{(-)}} \right)^3 \delta \rho^{(+)} + \delta \rho^{(-)} \right] \quad (18b)$$

Defining the potentials :

$$\varphi^{(+)} = \varepsilon \gamma_{00}^{(+)} \frac{c^2}{2} \quad \varphi^{(-)} = \varepsilon \gamma_{00}^{(-)} \frac{c^2}{2} \quad (19)$$

we get :

$$\frac{d^2 x^{(+)\alpha}}{dt^2} = - \frac{\partial \varphi^{(+)}}{\partial x_\alpha^{(+)}} \quad \frac{d^2 x^{(-)\alpha}}{dt^2} = - \frac{\partial \varphi^{(-)}}{\partial x_\alpha^{(-)}} \quad (20)$$

$$\sum_{\alpha=1}^3 \frac{\partial^2 \varphi^{(+)}}{\partial x^{(+)\alpha} \partial x_\alpha^{(+)}} = 4 \pi G \left[ \delta \rho^{(+)} + \left( \frac{a^{(-)}}{a^{(+)}} \right)^3 \delta \rho^{(-)} \right] \quad (21a)$$

$$\sum_{\alpha=1}^3 \frac{\partial^2 \varphi^{(-)}}{\partial x^{(-)\alpha} \partial x_\alpha^{(-)}} = -4 \pi G \left[ \left( \frac{a^{(+)}}{a^{(-)}} \right)^3 \delta \rho^{(+)} + \delta \rho^{(-)} \right] \quad (21b)$$

which are Poisson-like equations, associated to our bimetric model. Building the dynamics from this set of equations would perhaps require a more refined solution, involving different speeds of light  $c^{(+)}$  and  $c^{(-)}$ , and possibly time scale factors  $T^{(+)}$  and  $T^{(-)}$ . Anyway, it is possible to derive those laws in the simplified case  $a^{(+)} = a^{(-)}$ . Then :

$$\Delta \varphi^{(+)} = -\Delta \varphi^{(-)} = 4 \pi G (\delta \rho^{(+)} + \delta \rho^{(-)}) \quad (22)$$

$$\frac{d^2 x^\alpha}{dt^2} = - \frac{\partial \varphi^{(+)}}{\partial x_\alpha} \quad \frac{d^2 x^{(-)\alpha}}{dt^2} = - \frac{\partial \varphi^{(-)}}{\partial x_\alpha} = \frac{\partial \varphi^{(+)}}{\partial x_\alpha} \quad (23)$$

The subsequent interaction laws differ from GR's : *positive masses attract together through Newton's law. Negative masses attract together through Newton's law. Opposite signs masses repel each other through an anti-Newton's law.*

Referring to the previous works of H.Bondi [12] and G.W.Bonnor [28] the puzzling problem of runaway phenomenon is eliminated. By the way this model fits classical verification associated to GR. In effect, as the two species repel each other, where positive mass is present, negative mass is almost absent and the coupled field equations system reduces to :

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} ; \chi T_{\mu\nu}^{(+)} \quad (24a)$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} ; -\chi \left( \frac{a^{(+)}}{a^{(-)}} \right)^3 T_{\mu\nu}^{(+)} \quad (24b)$$

On another hand, considering portions of space containing a sphere filled by constant density matter (positive or negative), surrounded by void makes "internal" and "external" Schwarzschild solutions features clearer, as firstly presented in [18], introducing negative gravitational lensing effect, evoked in [23].

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