Lagrangian derivation of the two coupled field equations in the Janus cosmological model

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Abstract

After a review citing the results obtained in previous articles introducing the Janus Cosmological Model, consisting of a set of two coupled field equations, where one metrics refers to the positive masses and the other to the negative masses, which explains the observed cosmic acceleration and the nature of dark energy, we present the Lagrangian derivation of the model.

Keywords: Interacting positive and negative masses; Coupled field equations; Acceleration of the universe; Dark energy; Janus geometry; Janus cosmological model

1 Introduction

After F. Zwicky in 1931 and V. Rubin in 1979 pointing out the missing mass problem, the cosmological model was enriched with a new unidentified ingredient, the so called dark matter. Then a new problem appeared (Riess et al. 1998; Perlmutter et al. 1999; Riess 2000, 2004; Filippenko and Riess 2001; Leibindgut 2001; Knop et al. 2003; Tonry et al. 2003): instead of slowing down the universe was accelerating. Another distinct ingredient was added: dark energy. In Faraoni (2009), Faraoni et al. (2014) we quote:

> *–Dark energy and dark matter are the basic constituents of the universe.*

In this paper the authors, thinking about the interaction between those two, try to build a Lagrangian whose variation would give rise to the interacting equations. Following a completely different idea, Milgrom (1983, 1998), Milgrom and Sanders (2007) suggests an ad-hoc modification of Newton's law. But this last does not fit galactic clusters data. Nevertheless Combes (2015a,b) following Milgrom extends the idea to a set of four successive ad-hoc laws depending on distance (Combes 2015a,b). We quote:

> *–Gravity must be modified, adding a term, first at galactic scale, then another one to clusters of galaxies scale, and finally a third to very large scale–the one of dark energy.*

Negative mass related to negative pressure could explain the observed acceleration. Alas, in 1957 Bondi (1957) showed that the introduction of such ingredient in the Einstein's model produced unmanageable interaction laws. Einstein's equation, without cosmological constant, is:

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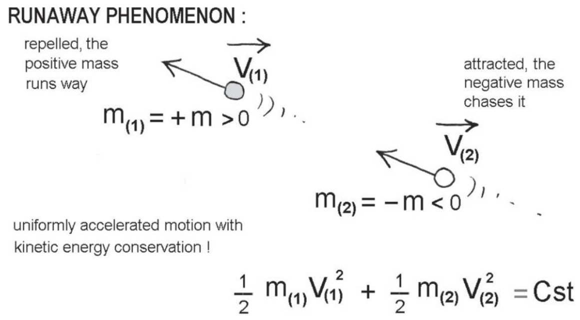

Fig. 1 The preposterous runaway phenomenon

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu} \tag{1}$$

When we apply Newtonian approximation (Adler et al. 1967, 10.5), expanding the metric into a series, from a Lorentz metric $g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon \gamma_{\mu\nu}$ we get Newton's law. Bondi writes:

$$m_{\mathrm{i}}^{(1)} \frac{\mathrm{d}^2 \vec{r}_1}{\mathrm{d}t^2} = \frac{G\,(\vec{r}_2 - \vec{r}_1)\, m_{\mathrm{p}}^{(1)} m_{\mathrm{a}}^{(2)}}{|\vec{r}_2 - \vec{r}_1|^3} \tag{2}$$

where $m_{(\mathrm{i})}$ is inertial mass, $m_{(\mathrm{p})}$ passive gravitational mass, and $m_{(\mathrm{a})}$ active gravitational mass. From equivalence principle $m_{(\mathrm{i})} = m_{(\mathrm{p})}$, so that:

$$\frac{\mathrm{d}^2 \vec{r}_1}{\mathrm{d}t^2} = \frac{G\,(\vec{r}_2 - \vec{r}_1)\, m_{\mathrm{p}}^{(1)} m_{\mathrm{a}}^{(2)}}{|\vec{r}_2 - \vec{r}_1|^3} \tag{3}$$

A consequence of (3) is that positive masses ($m_{(\mathrm{a})} > 0$) attract everything, while negative masses repel everything, which produces the preposterous runaway phenomenon: when a mass $+m$ encounters a mass $-m$, the first runs away, chased by the second. Both experience a uniform acceleration, while kinetic energy is saved, because one mass is negative, see Fig. 1. This analysis banned negative masses from cosmology during 57 years.

2 Back to "full geometry"

In a former paper (Petit and D'Agostini 2014a,b) we suggested to replace Einstein's equation by a set of two coupled field equations, which is a change of paradigm, which fits local verifications of RG, as shown in the following:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left(T_{\mu\nu}^{(+)} + \sqrt{\frac{-g^{(-)}}{-g(+)}}\, T_{\mu\nu}^{(-)} \right) \tag{4a}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left(\sqrt{\frac{-g^{(+)}}{-g(-)}}\, T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)} \right) \tag{4b}$$

where the tensors $T_{\mu\nu}^{(+)}$ and $T_{\mu\nu}^{(-)}$ are:

$$T_{\nu}{}^{(f)\mu} = \begin{pmatrix} \rho^{(f)} & 0 & 0 & 0 \\ 0 & -\frac{p^{(f)}}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{p^{(f)}}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{p^{(f)}}{c^2} \end{pmatrix} \quad \text{with} \begin{cases} \rho^{(f)} > 0 \text{ and } p^{(f)} > 0 & \text{for } f = \text{“}+\text{”} \\ \rho^{(f)} < 0 \text{ and } p^{(f)} < 0 & \text{for } f = \text{“}-\text{”} \end{cases} \tag{5}$$

Then Newtonian approximation:

$$
\begin{aligned}
g_{\mu\nu}^{(+)} &= \eta_{\mu\nu}^{(+)}(Lorentz) + \varepsilon\gamma_{\mu\nu}^{(+)} \\
g_{\mu\nu}^{(-)} &= \eta_{\mu\nu}^{(-)}(Lorentz) + \varepsilon\gamma_{\mu\nu}^{(-)}
\end{aligned}
\tag{6}
$$

provides completely different interaction laws:

– Positive masses mutually attract through Newton's law.

– Negative masses mutually attract through Newton's law.

– Masses with opposite signs mutually repel through "anti-Newton's law".

The present system was introduced in 1994 (Petit 1994). In 1995 (Petit 1995), 2D numerical simulations were performed. Assuming, after discoupling from radiation, that negative matter is denser, this last firstly forms clusters by gravitational instability, repelling positive matter in the remnant space, which forms a lacunar pattern, see Fig. 2. The whole is found very stable, the net of positive matter preventing negative clusters merging, while these last play the role of anchors, stabilizing the lacunar positive pattern.

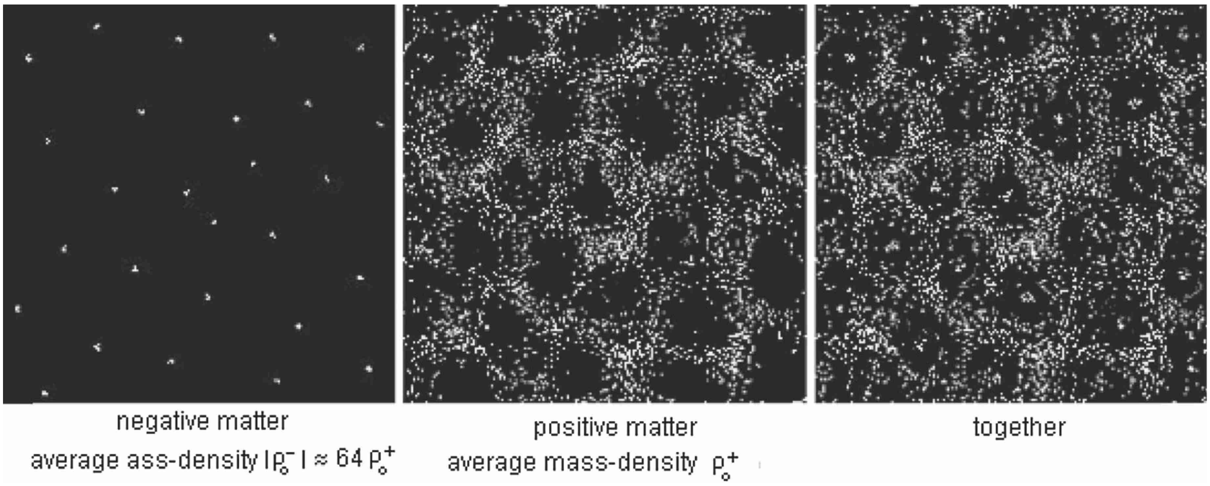

Fig. 2 2D simulation of Very Large Structure of the Universe (VLS), after Petit (1995)

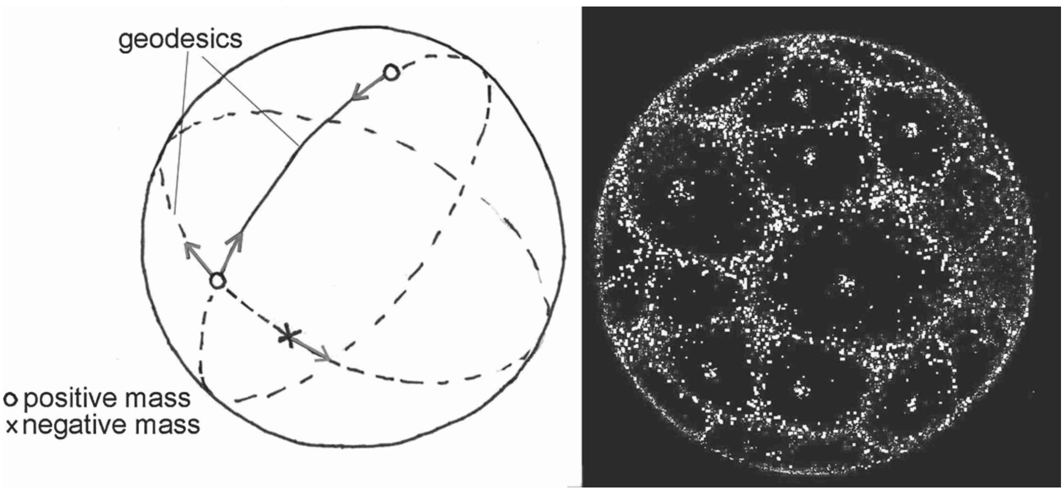

Fig. 3 Numerical 2D simulation of VLS in compact 2D workspace (Petit et al. 2015)

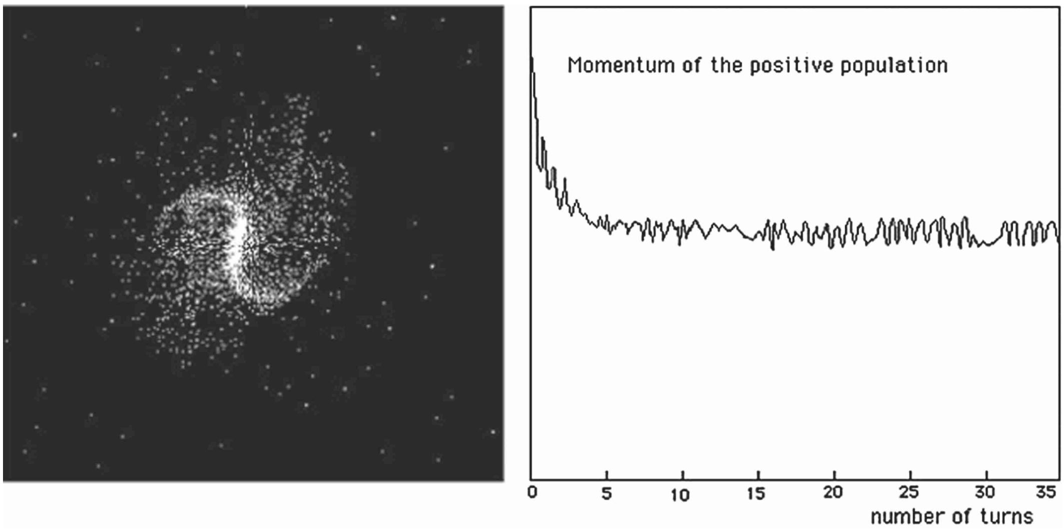

Fig. 4 Stable spiral structure, after Petit et al. (2001)

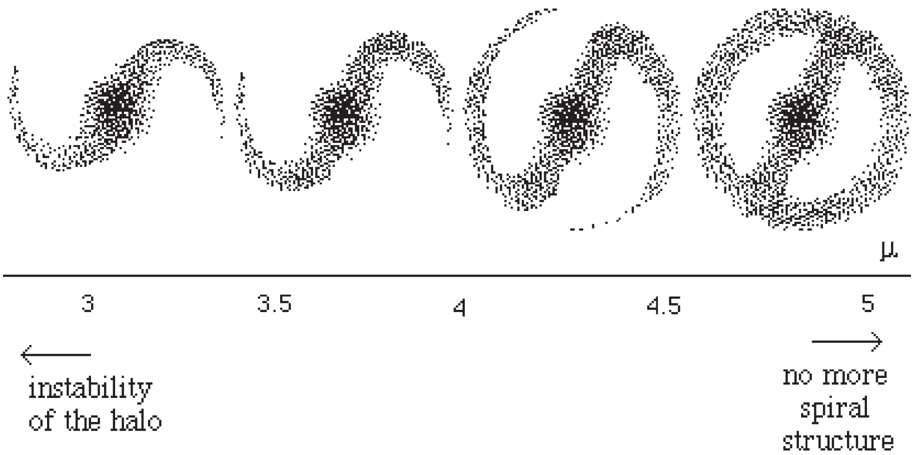

Fig. 5 Evolution du galactic design versus $\mu = \left|\frac{m^{(-)}}{m^{(+)}}\right|$ after Petit et al. (2001).

This theory has to be compared with recent results (Piran 1997; El-Ad et al. 1996, 1997; El-Ad and Piran 1997). Recently a new 2D simulation (Petit et al. 2015), performed on recent computers (5,000 positive mass points; 5,000 negative) introduced a compact 2D workspace, replacing euclidean distance, by distance measured along geodesics, see Fig. 3.

In Petit (1995) negative gravitational lensing effect was presented, which explained available data. The positive lensing is reinforced by the focusing action of negative matter environment, while this last explained galaxies' confinement. In 2000 another 2D simulation (Petit et al. (2001)) produced barred spirals, stable over 30 turns, the structure being due to dynamical friction with negative matter environment.

On the right of Fig. 4 we see a strong reduction of galaxy's momentum, due to dynamical friction, during five turns, which becomes almost negligible after that. Negative matter produces a potential barrier, which prevents spiral arms dissipation. Modifying the ratio negative mass/positive mass, different patterns are obtained, as shown in Fig. 5, which suggests a possible galaxies' evolution schema.

All these results were obtained on a personal computer. Collaboration with a group owing a fast computer could extend that work to 3D simulations, with compact workspace (S3 sphere).

Anyway, it is important to keep in mind that RG does not produce particles, only geodesics. The model is indeed a "pure geometrical description". As the model fits observation, we deduce that photons follow null-geodesics and matter follows non-null geodesics. In the Janus model, the second metric $g_{\mu\nu}^{(-)}$ produces a second, distinct, geodesic system. Similarly, we can assume that negative energy photons follow corresponding null-geodesics and negative masses follow corresponding non-null geodesics. Double Newtonian approximation (6) (around Lorentz Metrics) provides interaction schema. In (Petit and D'Agostini 2014a,b) an exact solution of system (4a)+(4b) for matter dominated era is built. Positive matter is found to accelerate, it obeys Bonnor's equation (Bonnor 1989). In the literature, dark matter and dark energy are considered as distinct (unknown) ingredients. In contrast, our new Janus Model takes into account all phenomena with a single "ingredient": negative mass. Its nature is cleared up through dynamic groups theory (Souriau 1970). The contents are similar: photons, electrons, protons, neutrons, and so on, with negative energies, and negative mass if the own one. In a recent publication (Petit and D'Agostini 2015) answers were given to a classical referee's critics: Encounters of opposite energy species are not possible, on geometrical grounds. Critic based on vacuum instability, due to $(+m, -m)$ pairs creation is also not relevant for such hypothetical quantum process is not described until now. In (Souriau 1970) the fact that negative energy ban was arbitrarily done in QFT (Weinberg 2005), when shifting to anti-unitary and anti-linear T operator (time inversion), was pointed out: unitary and linear T operator reverses energy.

Anyway, the system (4a)+(4b) needs a Lagrangian derivation. This is the subject of the present paper.

3 Lagrangian derivation of the JCM model. Janus geometry

In the following, we limit the method to $c^{(+)} = c^{(-)} = 1$ conditions, so that the determinants of the metrics are:

$$g^{(+)} = -\left(a^{(+)}\right)^6 \quad g^{(-)} = -\left(a^{(-)}\right)^6 \tag{7}$$

First, have a look on classical Lagrangian Einstein's equation derivation. A scalar density $L\sqrt{-g}$ is constructed (Adler et al. 1967, Eq. (11.110)), such that, under the variation of the metric field:

$$\delta \int_{D4} L\sqrt{-g}\,\mathrm{d}^4x = \int_{D4} T_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\,\mathrm{d}^4x \tag{8}$$

Then the equation of the gravitational field in a nonempty space can be expressed in the variational form (Adler et al. 1967, Eq. (11.111)):

$$\delta \int_{D4} (R - \chi L)\sqrt{-g}\,\mathrm{d}^4x = 0 \tag{9}$$

$(R - \chi L)\sqrt{-g}$ is the scalar density of this system.

The following relation holds:

$$\delta \int_{D^4} R\sqrt{-g}\,\mathrm{d}^4x = \int_{D^4} G_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\,\mathrm{d}^4x \tag{10}$$

where the Einstein tensor is defined by $G_{\mu\nu} = \left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)$.

Now let us consider the following scalar density, associated to a bimetric system:

$$\left(R^{(+)}\sqrt{-g^{(+)}} + R^{(-)}\sqrt{-g^{(-)}} - 2\chi L^{(+)}\sqrt{-g^{(+)}} - 2\chi L^{(-)}\sqrt{-g^{(-)}}\right) \tag{11}$$

Tensors $T_{\mu\nu}^{(+)}$ and $T_{\mu\nu}^{(-)}$ obey equations similar to (5):

$$\delta \int_{D4} L^{(+)} \sqrt{-g^{(+)}}\, \mathrm{d}^4 x = \int_{D4} T^{(+)}_{\mu\nu} \sqrt{-g^{(+)}} \delta g^{(+)\mu\nu}\, \mathrm{d}^4 x \tag{12a}$$

As $\rho^{(-)}$ and $p^{(-)}$ are negative, let us write:

$$\delta \int_{D4} L^{(-)} \sqrt{-g^{(-)}}\, \mathrm{d}^4 x = \int_{D4} -T^{(-)}_{\mu\nu} \sqrt{-g^{(-)}} \delta g^{(-)\mu\nu}\, \mathrm{d}^4 x \tag{12b}$$

Similarly, tensors $R^{(+)}_{\mu\nu}$ and $R^{(-)}_{\mu\nu}$ obey equations similar to (10):

$$\delta \int_{D^4} R^{(+)} \sqrt{-g^{(+)}}\, \mathrm{d}^4 x = \int_{D^4} G^{(+)}_{\mu\nu} \sqrt{-g^{(+)}} \delta g^{(+)\mu\nu}\, \mathrm{d}^4 x \tag{13a}$$

where $G^{(+)}_{\mu\nu} = \left(R^{(+)}_{\mu\nu} - \frac{1}{2} R^{(+)} g^{(+)}_{\mu\nu} \right)$

$$\delta \int_{D^4} R^{(-)} \sqrt{-g^{(-)}}\, \mathrm{d}^4 x = \int_{D^4} G^{(-)}_{\mu\nu} \sqrt{-g^{(-)}} \delta g^{(-)\mu\nu}\, \mathrm{d}^4 x \tag{13b}$$

where $G^{(-)}_{\mu\nu} = \left(R^{(-)}_{\mu\nu} - \frac{1}{2} R^{(-)} g^{(-)}_{\mu\nu} \right)$

Remark The integration refers to a 4D closed domain D_4. Either we consider that on the border the function under integration has a fixed value (zero). Either we can assume that the metrics are Lorentzian and the matter density is zero at infinite, which gives both $R^{(+)}$, $R^{(-)}$ and $L^{(+)}$, $L^{(-)}$ equal to zero.

Now let's build a bivariation in a bimetric functional space the metrics $g^{(+)}_{\mu\nu}$ and $g^{(-)}_{\mu\nu}$ belong to. A link between the two variations is required. Let's write:

$$\delta g^{(+)\mu\nu} = -\delta g^{(-)\mu\nu} \tag{14}$$

A question arises immediately: what could be the physical significance of such a link? The two metrics belong to a functional space. First, have a look on the external Schwarzschild metric (Schwarzschild 1916):

$$g_{\mu\nu} = \begin{pmatrix} \left(1 - \frac{2GM}{c^2 r}\right) c^2 & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \tag{15}$$

Such metric belongs to a metrics' subspace whose elements depend on a single parameter M. As described first in Petit (1994, 1995); Petit et al. (2001) the metric solutions $\left(g^{(+)}_{\mu\nu}, g^{(-)}_{\mu\nu}\right)$ of coupled field equations are joint solutions. For example, if an external Schwarzschild solution refers to a portion of empty space surrounding a positive mass $M^{(+)}$ we will write:

$$g^{(+)}_{\mu\nu} = \begin{pmatrix} \left(1 - \frac{2GM^{(+)}}{c^2 r}\right) c^2 & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1 - \frac{2GM^{(+)}}{c^2 r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \tag{16a}$$

associated to:

$$g^{(-)}_{\mu\nu} = \begin{pmatrix} \left(1 + \frac{2GM^{(+)}}{c^2 r}\right) c^2 & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1 + \frac{2GM^{(+)}}{c^2 r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \tag{16b}$$

with:

$$\frac{2GM^{(+)}}{c^2} = R_\mathrm{s} \text{ (Schwarzschild's radius} \tag{17}$$

and similar relation if these joint geometries depend on a negative mass $M^{(-)}$.
If $r \gg R$:

$$g^{(+)}_{\mu\nu} \simeq \begin{pmatrix} \left(1 - \frac{2GM^{(+)}}{c^2 r}\right) c^2 & 0 & 0 & 0 \\ 0 & 1 + \frac{2GM^{(+)}}{c^2 r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \tag{18a}$$

$$g^{(-)}_{\mu\nu} \simeq \begin{pmatrix} \left(1 + \frac{2GM^{(+)}}{c^2 r}\right) c^2 & 0 & 0 & 0 \\ 0 & 1 - \frac{2GM^{(+)}}{c^2 r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \tag{18b}$$

In the sub-space of the external Schwarzschild metrics:

$$\delta g^{(+)}_{\mu\nu} = \begin{pmatrix} -\frac{2G}{r} & 0 & 0 & 0 \\ 0 & +\frac{2G}{c^2 r} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \delta M^{(+)} \tag{19a}$$

$$\delta g^{(-)}_{\mu\nu} = \begin{pmatrix} \frac{2G}{r} & 0 & 0 & 0 \\ 0 & -\frac{2G}{c^2 r} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \delta M^{(+)} \tag{19b}$$

Similar calculation shows that $\delta g^{(+)\mu\nu} = \varphi(r)\delta M^{(+)}$, $\delta g^{(-)\mu\nu} = -\varphi(r)\delta M^{(+)}$. Then the coupled metrics obey (14).

An external Schwarzschild's metric element is "organized" around some mass concentration M. For example, if we neglect the planetary system, the geometry of the solar system depends on a single parameter, the mass M of the Sun.

During the matter-dominated era, the distribution of matter is not described by a continuous distribution of mass. Matter is located in relatively small and finite portions of space, surrounded by void, which is consistent with relationship (14).

Anyway we will consider Eq. (14) as the mathematical definition of Janus geometry.

Then, to the above Eqs. (12a), (12b) and (13a), (13b), we can add:

$$\delta \int_{D4} L^{(+)} \sqrt{-g^{(+)}}\, \mathrm{d}^4 x = - \int_{D4} T^{(+)}_{\mu\nu} \sqrt{\frac{-g^{(+)}}{-g^{(-)}}} \sqrt{-g^{(-)}} \delta g^{(-)\mu\nu}\, \mathrm{d}^4 x \tag{20a}$$

$$\delta \int_{D4} L^{(-)} \sqrt{-g^{(-)}}\, \mathrm{d}^4 x = + \int_{D4} T^{(-)}_{\mu\nu} \sqrt{\frac{-g^{(-)}}{-g^{(+)}}} \sqrt{-g^{(+)}} \delta g^{(+)\mu\nu}\, \mathrm{d}^4 x \tag{20b}$$

Write

$$\delta \int_{D4} \left(R^{(+)} \sqrt{-g^{(+)}} + R^{(-)} \sqrt{-g^{(-)}} - 2\chi L^{(+)} \sqrt{-g^{(+)}} - 2\chi L^{(-)} \sqrt{-g^{(-)}} \right) \mathrm{d}^4 x = 0 \tag{21}$$

Combining (12a), (12b), (13a), (13b), (20a), (20b) with (21) we find:

$$\int_{D^4} \left\{ \left[R^{(+)}_{\mu\nu} - \frac{1}{2} R^{(+)} g^{(+)}_{\mu\nu} - \chi \left(T^{(+)}_{\mu\nu} + \sqrt{\frac{-g^{(-)}}{-g^{(+)}}} T^{(-)}_{\mu\nu} \right) \right] \sqrt{-g^{(+)}} \delta g^{(+)\mu\nu} \right.$$
$$\left. + \left[R^{(-)}_{\mu\nu} - \frac{1}{2} R^{(-)} g^{(-)}_{\mu\nu} - \chi \left(\sqrt{\frac{-g^{(+)}}{-g^{(-)}}} T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu} \right) \right] \sqrt{-g^{(+)}} \delta g^{(+)\mu\nu} \right\} \mathrm{d}^4 x = 0 \quad (22)$$

Which is satisfied by the following system:

$$R^{(+)}_{\mu\nu} - \frac{1}{2} R^{(+)} g^{(+)}_{\mu\nu} = \chi \left(T^{(+)}_{\mu\nu} + \sqrt{\frac{-g^{(-)}}{-g^{(+)}}} T^{(-)}_{\mu\nu} \right) \quad (23a)$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2} R^{(-)} g^{(-)}_{\mu\nu} = -\chi \left(\sqrt{\frac{-g^{(+)}}{-g^{(-)}}} T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu} \right) \quad (23b)$$

with

$$\chi = -\frac{8\pi G}{c^2} \quad (24)$$

If one assumes that the two speeds of light $c^{(+)}$ and $c^{(-)}$ are equal, we can set $c^{(+)} = c^{(1)} = 1$ and find again the system (1a)+(1b) which has been introduced and studied in Petit and D'Agostini (2014a,b).

For $c^{(+)} \neq c^{(-)}$ conditions, the paper (Petit and D'Agostini 2014a,b) gives more details.

4 Physical features from Janus geometry

In reference Petit and D'Agostini (2014a,b), we have built the Newtonian approximation. Then interaction laws and behavior arise, different from Bondi (1957) and Bonnor (1989).

- Masses with same signs mutually attract along Newton's law
- Masses with opposite signs mutually repel along "Anti-Newton's" law

Simulations have shown that the two populations separate by Petit (1995). This phenomenon agrees with Piran (1997), El-Ad et al. (1996, 1997), El-Ad and Piran (1997).

Time-dependent exact solution (Petit and D'Agostini 2014a,b) agrees with Riess et al. (1998), Perlmutter et al. (1999), Riess (2000); Filippenko and Riess (2001), Leibindgut (2001), Knop et al. (2003), Tonry et al. (2003), Riess (2004).

The link between mass inversion and time inversion (Souriau 1970) was presented in Petit et al. (2001).

The model fits local observational data. Indeed, negative matter is negligible around the Sun so that the system reduces to:

$$R^{(+)}_{\mu\nu} - \frac{1}{2} R^{(+)} g^{(+)}_{\mu\nu} \approx \chi T^{(+)}_{\mu\nu} \quad (25a)$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2} R^{(-)} g^{(-)}_{\mu\nu} \approx -\chi \sqrt{\frac{-g^{(+)}}{-g^{(-}}} T^{(+)}_{\mu\nu} \quad (25b)$$

(25a) identifies to Einstein's equation. Conversely at the center of big voids of Very Large Structure positive matter is negligible and negative matter dominates. Same thing between galaxies, so that we have there:

$$R^{(+)}_{\mu\nu} - \frac{1}{2}R^{(+)}g^{(+)}_{\mu\nu} \approx \chi\sqrt{\frac{-g^{(-)}}{-g^{(+}}}T^{(-)}_{\mu\nu} \tag{26a}$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu} \approx -\chi T^{(-)}_{\mu\nu} \tag{26b}$$

This corresponds to an "induced geometry", due to the action of negative mass on positive mass and positive energy photons paths (negative gravitational lensing, firstly presented by Petit 1995).

5 Conclusion

Based on Janus condition, defining Janus geometry (14), Lagrangian derivation of the set of two field equations corresponding to our Janus Cosmological Model (JCM) is presented. Observational agreement is detailed. Slightly similar works from other authors correspond to Hossenfelder (2008) and Henry-Couannier (2005). In the paper (Petit and D'Agostini 2014a,b) the model explains the acceleration of the Universe and suggests the nature of so-called dark energy. The paper (Petit and D'Agostini 2014a,b) deals with different speeds of light, for positive and negative particles. A future paper to be published is devoted to radiation dominated era and to variable constants regime that will explain $c^{(-)} > c^{(+)}$.

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