Energy gain of runaway electrons in vertical disruptions

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- introduction
- energy conversion - qualitative picture
- 2D model - setup and equations
- numerical results
- summary

Introduction (1)

- **runaway electrons (REs):**
  - **electrons accelerated to relativistic speeds**
  - **energy gain exceeds energy loss**
  - **experience non-monotonic friction force** $\mathbf{F}(v)$
- **RE formation mechanisms:**
  - **primary: Dreicer, hot-tail,** γ (not discussed here)
  - **secondary: avalanche for** $E > E_\mathrm{c}$ (important here)
    E_c - critical field strength (required to maintain current of REs)
  - **REs likely to be generated during tokamak disruptions**
- **REs are a possible threat to future tokamaks:**
  - **strongly localized beams with high energies** $(\sim \mathrm{MeV})$
  - **deep penetration of materials** $(\sim \mathrm{cm})$
  - **more REs for large devices expected** $(G_\mathrm{RE} \sim \exp 2.5\, I_\mathrm{P}(\mathrm{MA}))$

Example: REs in Tore Supra

investigation of REs on Tore Supra:

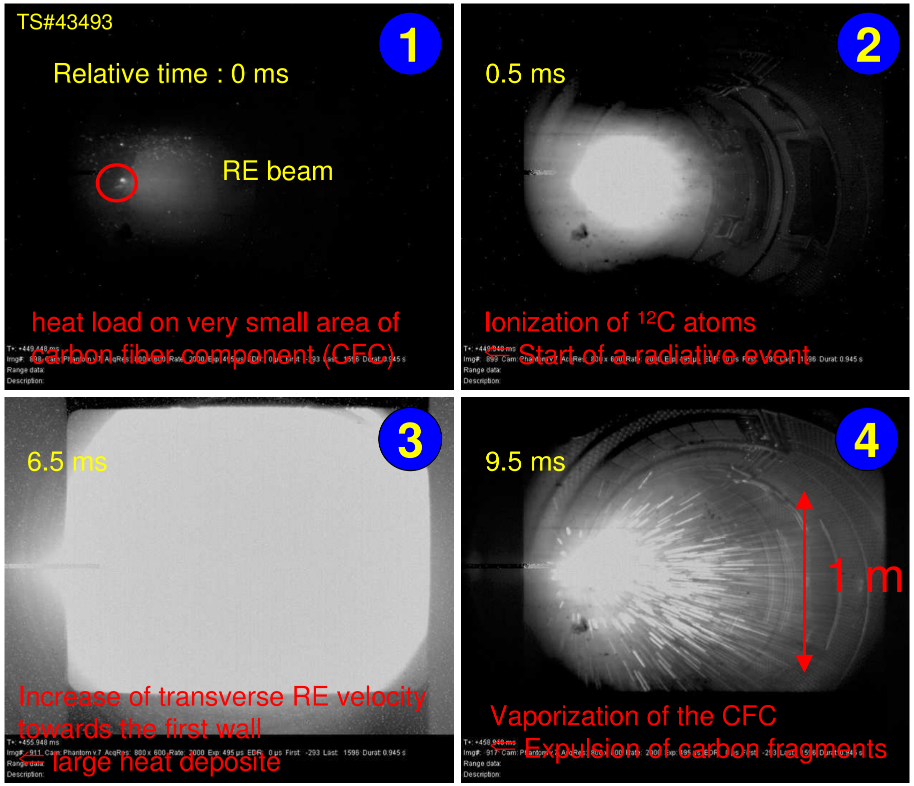
- "slowing-down of REs takes time ... "
- active current and position control
- RE plateaus controlled for some seconds
- massive gas injection applied
- part of disruption mitigation strategy
- sudden RE current collapses observed

F. Saint-Laurent et al.

Control of runaway electron heat loads on Tore Supra

38$^{\text{th}}$ EPS conference, Strasbourg (2011)

60 kA runaway electron beam striking CFC wall in Tore Supra



courtesy of F. Saint-Laurent

Introduction (2)

- theoretical & experimental studies
  - RE formation and properties
  - energy conversion by REs (*Putvinski et al., Loarte et al.*)
  - RE loss mechanisms
- strategies for RE suppression or mitigation wanted
  - collisional slowing down (killer-pellets, massive gas injection, ...)
  - drift orbit losses through RMPs $\Longrightarrow$ *G. Papp (O-26)*
  - RE current control (*Saint-Laurent et al.*)
- RE control closely connected to disruption control

$\Longrightarrow$ **REs will be an important issue for ITER operation!**

Possible Conditions after Current Quench in ITER

- **population of REs with** $\sim 10\,\mathrm{MeV}$
- **current conversion of up to** $\sim 2/3\, I_\mathrm{P}^0$ **possible**
  - **initial RE current** $I_\mathrm{RE} \sim 10\,\mathrm{MA}$
  - **initial RE density** $n_\mathrm{RE} \sim 10^{16}\,\mathrm{m}^{-3}$ ($N_\mathrm{RE} \sim 10^{19}$)
- **background plasma with** $T \sim 5\,\mathrm{eV}$
  - **determines free electron density** $n_\mathrm{e} \sim 10^{21}\,\mathrm{m}^{-3}$
  - **no significant contribution to current (high resistivity)**
- **initial kinetic energy of REs** $W_\mathrm{RE}^0 \sim 20\,\mathrm{MJ}$
  - **small compared to magnetic field energy!**

$$\frac{W_\mathrm{RE}^0}{W_\mathrm{m}^\mathrm{pol}} \sim \frac{(\gamma - 1) I_\mathrm{A}}{I_\mathrm{P}^0} \sim 0.03$$

- **instabilities causing the plasma to move toward the wall**

Energy Conversion - Qualitative Picture

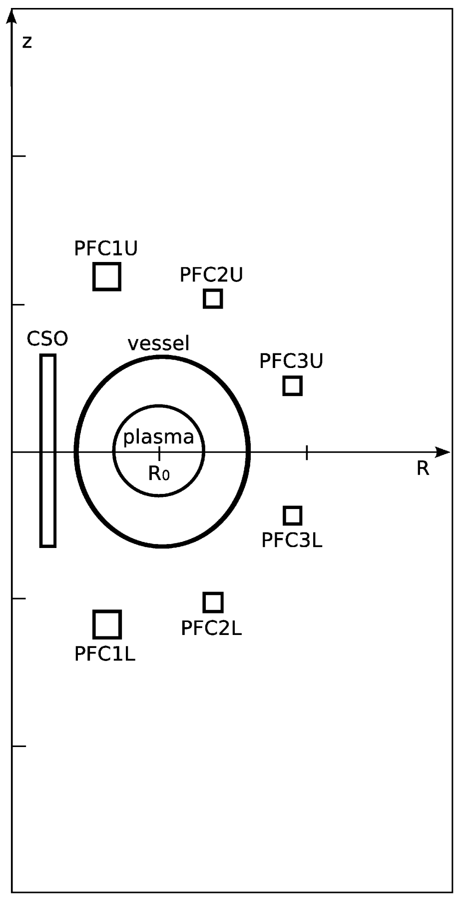
what will happen:

- **plasma drifts toward walls and induces eddy currents**
- **back reaction on plasma controls motion**
- **plasma hits wall and is getting scraped off**
- **rapid current loss causes strong toroidal fields driving REs**
- **amplification of REs at cost of poloidal field energy**
- **energy of poloidal field dissipated in plasma and walls**

Energy Conversion in Fusion Plasmas with REs

- *Putvinski et al., Plas. Phys. Contr. Fusion* **39** *(1997)*
  - poloidal magnetic field is reservoir of free energy
  - strong amplification of RE energy during vertical drift possible
  - 1D model for straight plasma cylinder enclosed by cylindrical wall
  - highest RE wall loads predicted for slow disruptions
- *Loarte et al., Nucl. Fusion* **51** *073004 (2011)*
  - experimental evidence for RE energy conversion on JET
  - 1D numerical simulation results
- next step: 2D modelling (axisymmetric)
  - plasma with circular cross section
  - self-consistent vertical motion of plasma
  - resistive diffusion in conducting structures external to plasma

2D Computational Model Setup



domain with objects

Model assumptions:

- I_{P}^0 carried by REs exclusively
- circular plasma cross section
- large aspect ratio
- up-down symmetric objects
- up-down symmetric PF1 coil current ($\uparrow\uparrow I_{\mathrm{P}}$)
- no other PF coil/CSO currents applied
- vertical motion only (plasma rest frame)

Numerics:

- FV ansatz and Newton's method
- $\psi = 0$ at $R = \varepsilon$, $\nabla\psi \cdot d\mathbf{S} = 0$ elsewhere
- non-equidistant grid (finest at plasma center)

2D-Mathematical Model

- **magnetic field in axisymmetric geometry**

$$\mathbf{B} = I(\psi, t)\nabla\varphi + \nabla\varphi \times \nabla\psi$$

$\psi(R, z, t) \sim$ poloidal magnetic flux

- **toroidal current density (runaway + Ohmic)**

$$J_\varphi = J_\mathrm{r} + \sigma E_\varphi = \frac{\Delta^* \psi}{\mu_0 R} \qquad \left(\Delta^* = R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial}{\partial R} \right)$$

- **Grad-Shafranov-like equation in plasma and external conductors**

$$\sigma \mu_0 \frac{\partial \psi}{\partial t} = \Delta^* \psi - \mu_0 R J_\mathrm{r} - \underbrace{\sigma \mu_0 v_z \frac{\partial \psi}{\partial z}}_{\text{objects moving with } v_z}$$

solve for time evolution of poloidal magnetic field!

Evolution of Runaway Current

$$\frac{1}{J_\mathrm{r}}\frac{\partial J_\mathrm{r}}{\partial t} \simeq \left(\frac{\langle E_\varphi\rangle}{E_\mathrm{c}} - 1\right)\left(\frac{\Theta(\langle E_\varphi\rangle - E_\mathrm{c})}{\tau_\mathrm{a}} + \frac{\Theta(E_\mathrm{c} - \langle E_\varphi\rangle)}{\tau_\mathrm{d}}\right)$$

E_c - critical field strength

- **exponential growth for $\langle E_\varphi\rangle > E_\mathrm{c}$ by avalanche**
  unconventional Ohm's law with avalanche time

$$\tau_\mathrm{a} = \tau \ln\Lambda \sqrt{\frac{3(Z_\mathrm{eff}+5)}{\pi\gamma(\epsilon)}}\left(1 - \frac{E_\mathrm{c}}{E} + \frac{4\pi(Z_\mathrm{eff}+1)^2}{3\gamma(\epsilon)(Z_\mathrm{eff}+5)(E^2/E_\mathrm{c}^2 + 4/\gamma^2(\epsilon) - 1)}\right)^{1/2}$$

$\gamma = (1 + 1.46\sqrt{\epsilon} + 1.72\epsilon)^{-1}$, $\epsilon = r/R$, τ_a for $E \gg E_\mathrm{c}$

- **exponential decay for $\langle E_\varphi\rangle < E_\mathrm{c}$ by collisional damping**
  $\tau_\mathrm{d} = 2\tau\ln\Lambda$ - electron collisional damping time

- **radiation losses unimportant on time scales encountered here**

Self-Consistent Motion of Plasma

- **plasma inertia is very small $\Longrightarrow$ vertical force on plasma should vanish!**

$$F_z = \int\limits_{V_{\rm plas}} (\mathbf{J} \times \mathbf{B}) \cdot \nabla z \, dV \stackrel{!}{=} 0$$

- **with $\mathbf{J} = \mathbf{J}^{\rm in} + \mathbf{J}^{\rm ex}$ and $\mathbf{B} = \mathbf{B}^{\rm in} + \mathbf{B}^{\rm ex}$ there is** (for large aspect ratio)**:**

$$F_z = \int\limits_{V_{\rm plas}} \left(J_R^{\rm in} B_\varphi^{\rm ex} - J_\varphi^{\rm in} B_R^{\rm ex} \right) \, dV \simeq - \int\limits_{V_{\rm plas}} J_\varphi^{\rm in} B_R^{\rm ex} \, dV + O\left(\epsilon J_R^{\rm in} B_\varphi^{\rm ex} \right)$$

- **iterative procedure to determine v_z for time step Δt :**

move plasma as to obey condition $F_z(t + \Delta t) = 0$!

Energy Transfer to Runaway Electrons

- **total energy transferred to plasma** (background and runaway electrons)

$$W_{\text{plas}} = \int_0^t dt' \int_{V_{\text{plas}}} J_\varphi E_\varphi \, dV = \int_0^t dt' \int_{V_{\text{plas}}} (\sigma E_\varphi + J_{\text{r}})\, E_\varphi \, dV = W_\Omega + W_{\text{r}}$$

- **energy lost by REs through collisional slowing-down on background**

$$W_{E_{\text{c}}} \approx \int_0^t dt' \int_{V_{\text{plas}}} J_{\text{r}} E_{\text{c}} \, dV$$

- **final kinetic RE energy** (... which is going to strike the first wall!)

$$W_{\text{RE}} = W_{\text{RE}}^0 + \int_0^t dt' \int_{V_{\text{plas}}} J_{\text{r}}(E_\varphi - E_{\text{c}})\, dV \; = W_{\text{RE}}^0 + W_{\text{r}} - W_{E_{\text{c}}}$$

Numerical Results - Plasma Motion

A reference case

$I_P^0 = 10\,\mathrm{MA}$ (flat profile)

$T \simeq 5\,\mathrm{eV}$

$n \simeq 10^{21}\,\mathrm{m}^{-3}$

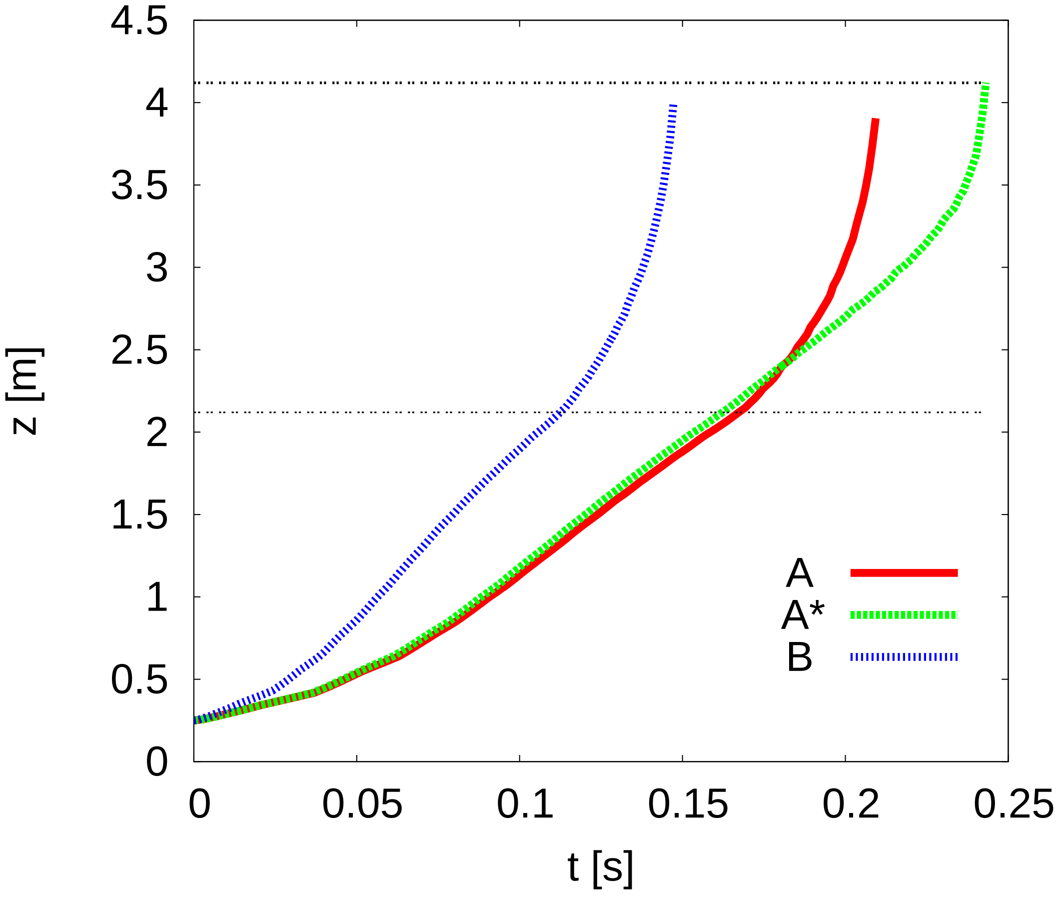
$I_{PF1} = 0.84\, I_P^0$

A* I_P^0 with peaked profile

B $I_{PF1} = 1.25\, I_P^0$

- initial displacement $\delta z = .25\,\mathrm{m}$
- free-motion phase $z < 2.1\,\mathrm{m}$
- plasma hits wall at $z = 2.1\,\mathrm{m}$
- scrape-off phase $z > 2.1\,\mathrm{m}$
- plasma depleted at $z = 4.1\,\mathrm{m}$

vertical position of plasma center $z(t)$



Numerical Results - RE Energy Gain vs. Time

A reference case

$I_P^0 = 10\,\mathrm{MA}$ (flat profile)

$T \simeq 5\,\mathrm{eV}$

$n \simeq 10^{21}\,\mathrm{m}^{-3}$

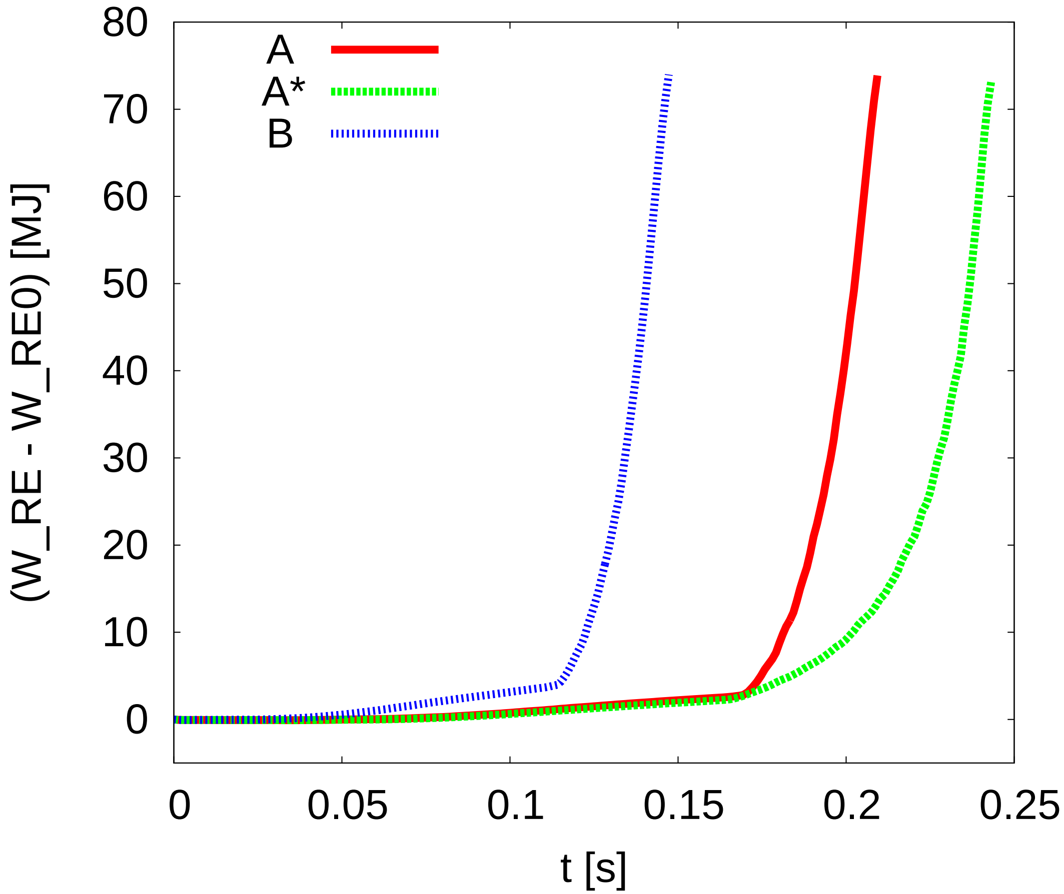
$I_{PF1} = 0.84\,I_P^0$

A* I_P^0 with peaked profile

B $I_{PF1} = 1.25\,I_P^0$

- growth in scrape-off phase
- $W_{RE}^0 \sim 20\,\mathrm{MJ}$
- total $W_{RE} \sim 100\,\mathrm{MJ}$ possible
- final values identical

kinetic energy gained by REs $(W_{RE} - W_{RE}^0)$ vs. t



Numerical Results - RE Energy Gain vs. Position

A reference case

$I_P^0 = 10\,\mathrm{MA}$ (flat profile)

$T \simeq 5\,\mathrm{eV}$

$n \simeq 10^{21}\,\mathrm{m}^{-3}$

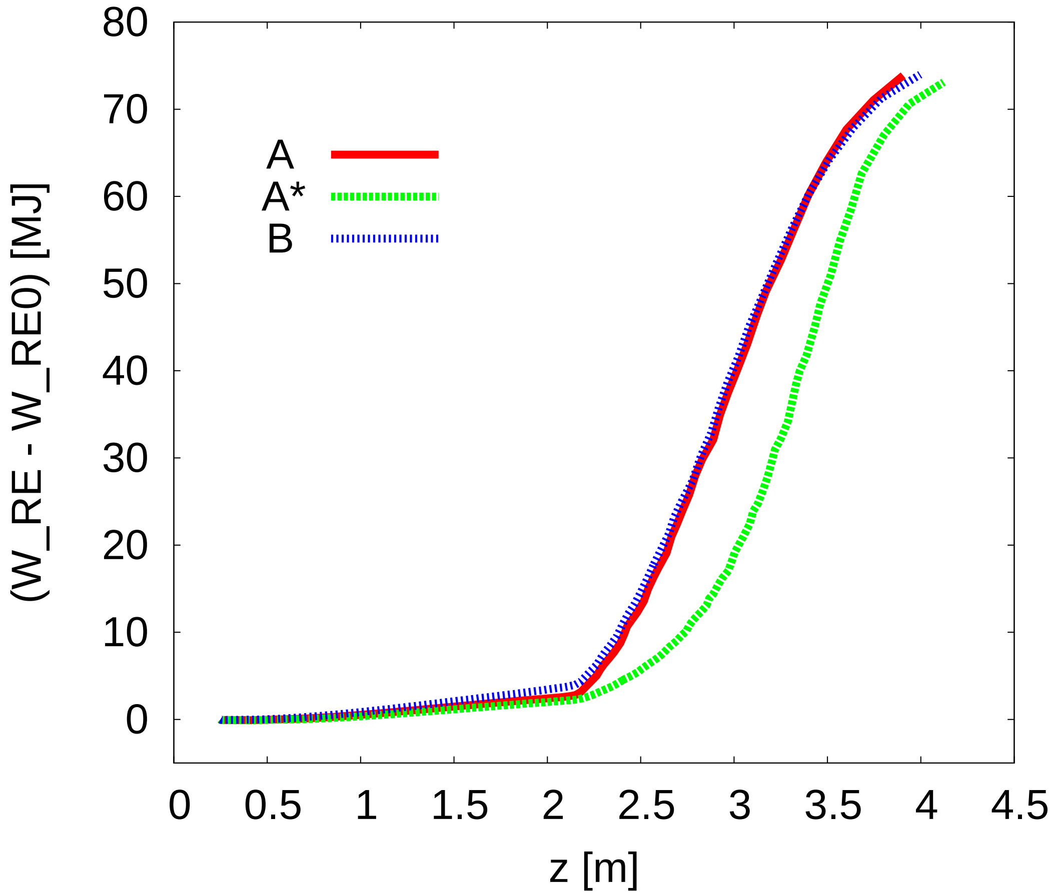
$I_{PF1} = 0.84\,I_P^0$

A* I_P^0 with peaked profile

B $I_{PF1} = 1.25\,I_P^0$

- growth in scrape-off phase
- $W_{RE}^0 \sim 20\,\mathrm{MJ}$
- total $W_{RE} \sim 100\,\mathrm{MJ}$ possible
- final values identical

kinetic energy gained by REs $(W_{RE} - W_{RE}^0)$ vs. z



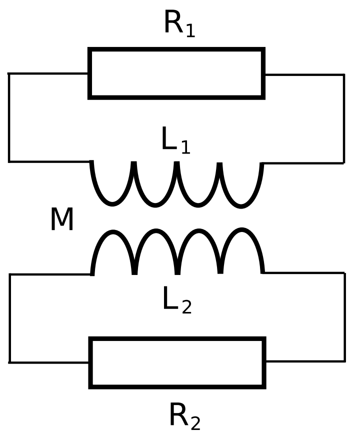
Energy Conversion in Fusion Plasmas with REs

- *Putvinski et al., Plas. Phys. Contr. Fusion* **39** *(1997)*
  $\Longrightarrow$ **highest RE wall loads predicted for slow disruptions**
- **supported by simple 2-circuit model:**

let currents be:
$I_1 = I_0 e^{-t/t_0}$ (plasma) and $I_2(t=0) = 0$ (wall)

voltages then:
$U_1 = -L_1 \dot{I}_1 - M \dot{I}_2$
$U_2 = R_2 I_2 = -L_2 \dot{I}_2 - M \dot{I}_1$

dissipated energies:
$W_1 = \left(L_1 - \frac{M^2}{(R_2 t_0 + L_2)}\right) \frac{I_0^2}{2} d \qquad W_2 = \frac{M^2}{(R_2 t_0 + L_2)} \frac{I_0^2}{2}$



$\omega t_0 = R_2 / L_2 \, t_0 \gg 1 \quad \Longrightarrow \quad W_1 \approx W_1 + W_2 = \frac{L_1 I_0^2}{2}$

- **energy gain for cases A and B found identical $\Longrightarrow$ contradiction?**

Numerical Results - Energy Contributions

A reference case

$I_P^0 = 10\,\mathrm{MA}$ (flat profile)

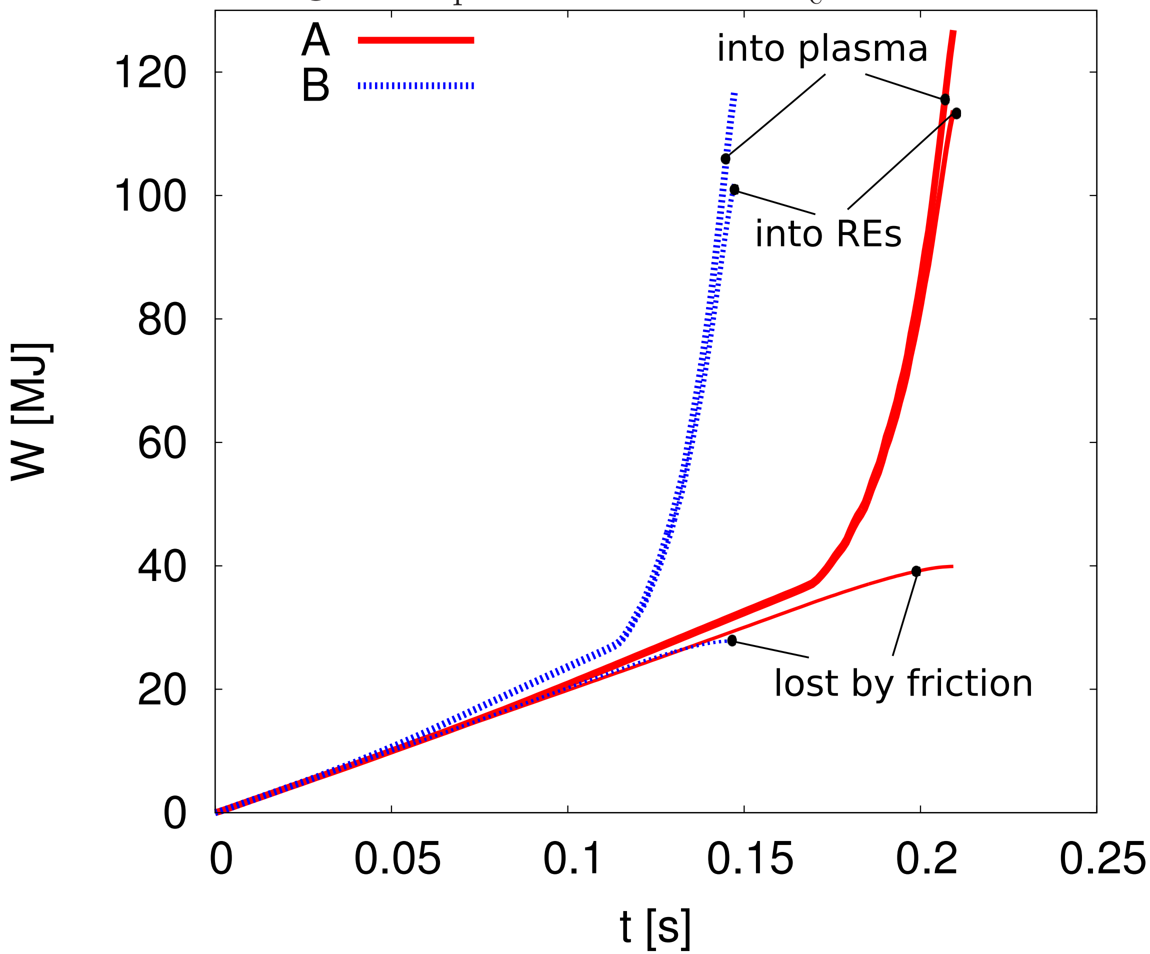
$T \simeq 5\,\mathrm{eV}$

$n \simeq 10^{21}\,\mathrm{m}^{-3}$

$I_{PF1} = 0.84\,I_P^0$

B $I_{PF1} = 1.25\,I_P^0$

energies W_{plas}, W_r and W_{E_c} vs. t



- total energy transfer $W_{plas} \sim t$
- friction (W_{E_c}) not negligible!
- ΔW_{RE} identical for A and B
- in free motion $W_{plas} \sim W_{E_c}$
- implications for E_φ?

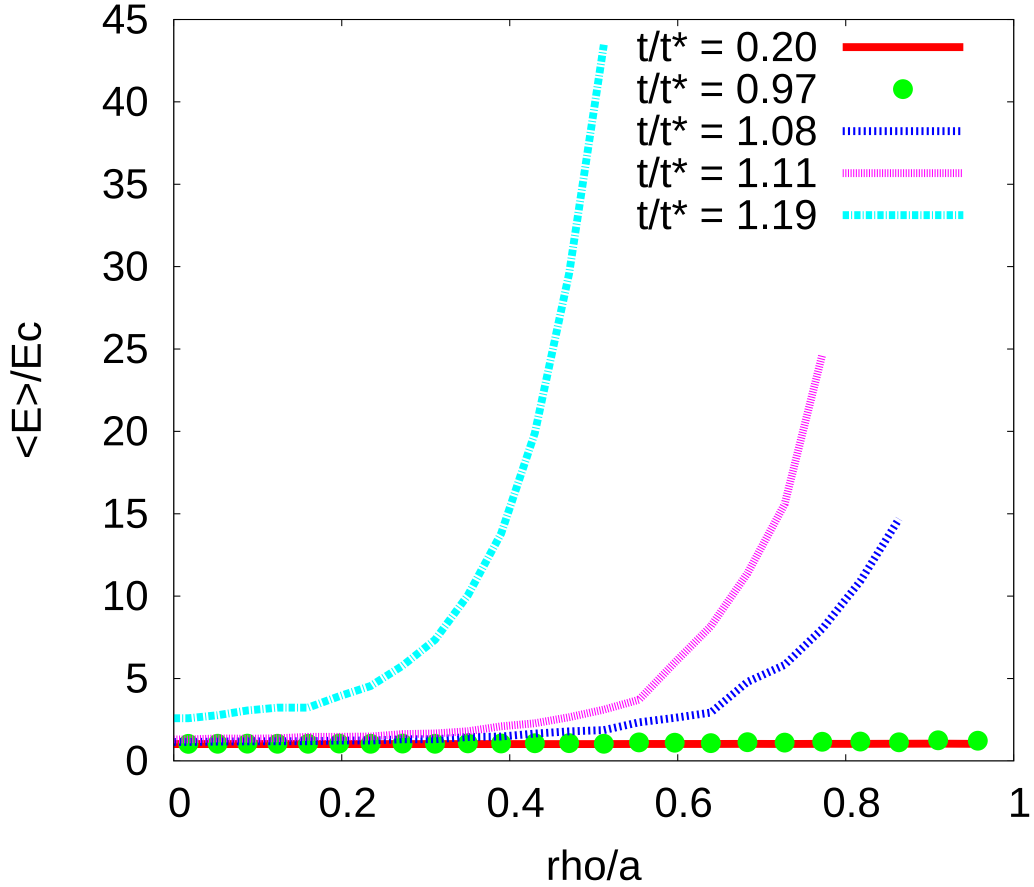
Numerical Results - Electric Field Strength

- reference case
  $I_\mathrm{P}^0 = 10\,\mathrm{MA}$ (flat profile)
  $T \simeq 5\,\mathrm{eV}$
  $n \simeq 10^{21}\,\mathrm{m}^{-3}$
  $I_\mathrm{PF1} = 0.84\,I_\mathrm{P}^0$

  plasma hits wall at $t = t^*$

- **free-motion phase** $(t < t^*)$**:**
  - motion makes $\langle E_\varphi \rangle \approx E_\mathrm{c}$
- **scrape-off phase** $(t > t^*)$**:**
  - $\langle E_\varphi \rangle \gg E_\mathrm{c}$ at edge $\Rightarrow$ REs!
  - $\langle E_\varphi \rangle \approx E_\mathrm{c}$ in center
  - shrinking plasma radius

electric field strength $\langle E_\varphi \rangle / E_\mathrm{c}$ vs. (ϱ/a)



Electric Field Strength Estimate

$\langle E_\varphi \rangle \approx E_\mathrm{c}$ in most of the plasma is characteristic feature!

- there must be (displacement current neglected):

$$\nabla^2 E = \mu_0 \frac{\partial J_\varphi}{\partial t} \qquad \text{with} \quad J_\varphi = J_\mathrm{r} \quad \text{and} \quad \frac{\partial J_\mathrm{r}}{\partial t} \simeq \frac{J_\mathrm{r}}{\tau_\mathrm{a}} \left(\frac{E}{E_\mathrm{c}} - 1 \right)$$

- if $J_\varphi = J_\mathrm{r}$ with $\partial J_\mathrm{r} / \partial t \simeq J_\mathrm{r} \left(\frac{E}{E_\mathrm{c}} - 1 \right) / \tau_\mathrm{a}$
- then:

$$a^2 \nabla^2 E = \frac{a^2 \mu_0 J_\mathrm{r}}{\tau_\mathrm{a} E_\mathrm{c}} (E - E_\mathrm{c})$$

- estimate r.h.s.:

$$\frac{a^2 \mu_0 J_\mathrm{r}}{\tau_\mathrm{a} E_\mathrm{c}} (E - E_\mathrm{c}) \sim \frac{\mu_0 I_\mathrm{r}}{\pi \tau_\mathrm{a} E_\mathrm{c}} (E - E_\mathrm{c}) \approx \frac{I_\mathrm{r}}{0.2\,\mathrm{MA}} (E - E_\mathrm{c})$$

- since $I_\mathrm{r} \gg 0.2\,\mathrm{MA}$ there follows $E \approx E_\mathrm{c}$ or $(a^2 \nabla^2 E)/E \gg 1$ (plasma edge)

$\Longrightarrow$ **use full expression for τ_a!**

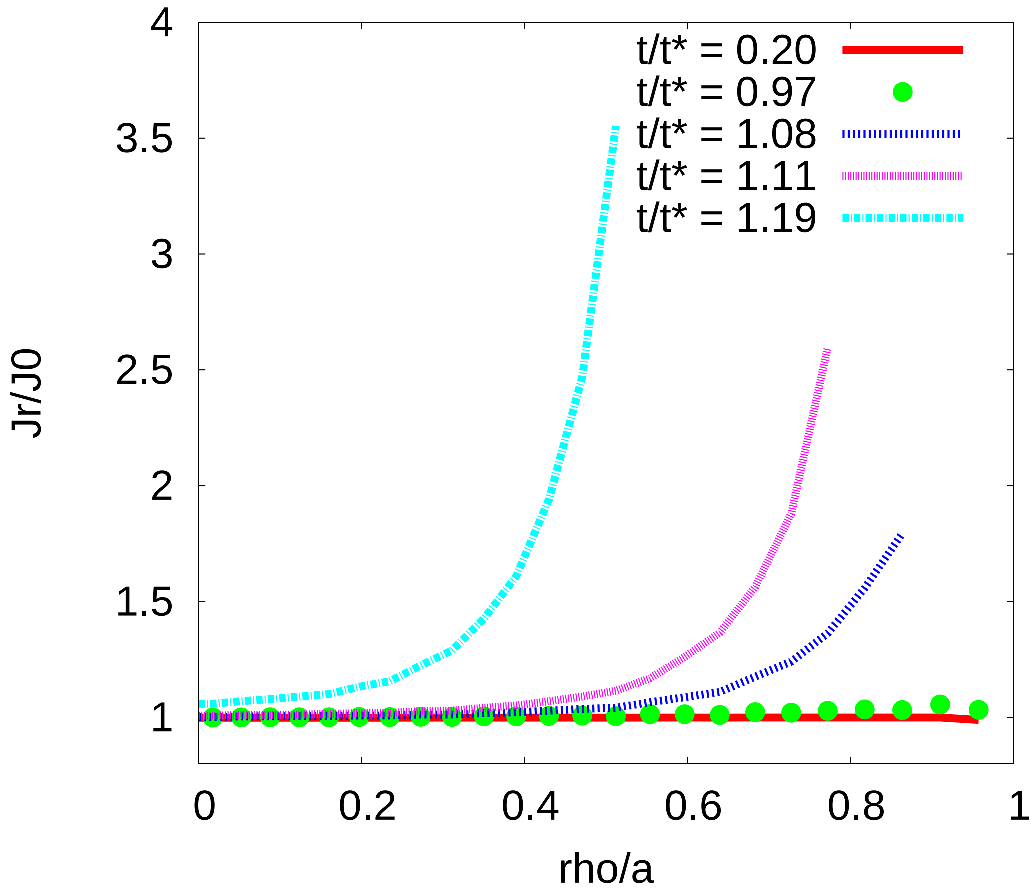
Numerical Results - Current Density

- reference case
  $I_{\rm P}^0 = 10\,{\rm MA}$ (flat profile)
  $T \simeq 5\,{\rm eV}$
  $n \simeq 10^{21}\,{\rm m}^{-3}$
  $I_{\rm PF1} = 0.84\,I_{\rm P}^0$

  plasma hits wall at $t = t^*$

- RE amplification at edge
- hollow current profile
- skin depth $\lambda \sim 0.1\,a$ (Putvinski)

RE current density $J_{\rm r}/J_0$ vs. (ϱ/a)



Numerical Results - Parameter Dependence

A reference case

$I_P^0 = 10\,\mathrm{MA}$ (flat profile)

$T \simeq 5\,\mathrm{eV}$

$n \simeq 10^{21}\,\mathrm{m}^{-3}$

$I_{\mathrm{PF1}} = 0.84\,I_P^0$

C $T^{\mathrm{C}} = 2\,T^{\mathrm{A}}$

$\Rightarrow$ $\tau_{\mathrm{skin}}^{\mathrm{plas}}$ larger

D $n^{\mathrm{D}} = 2\,n^{\mathrm{A}}$

$\Rightarrow$ E_c higher

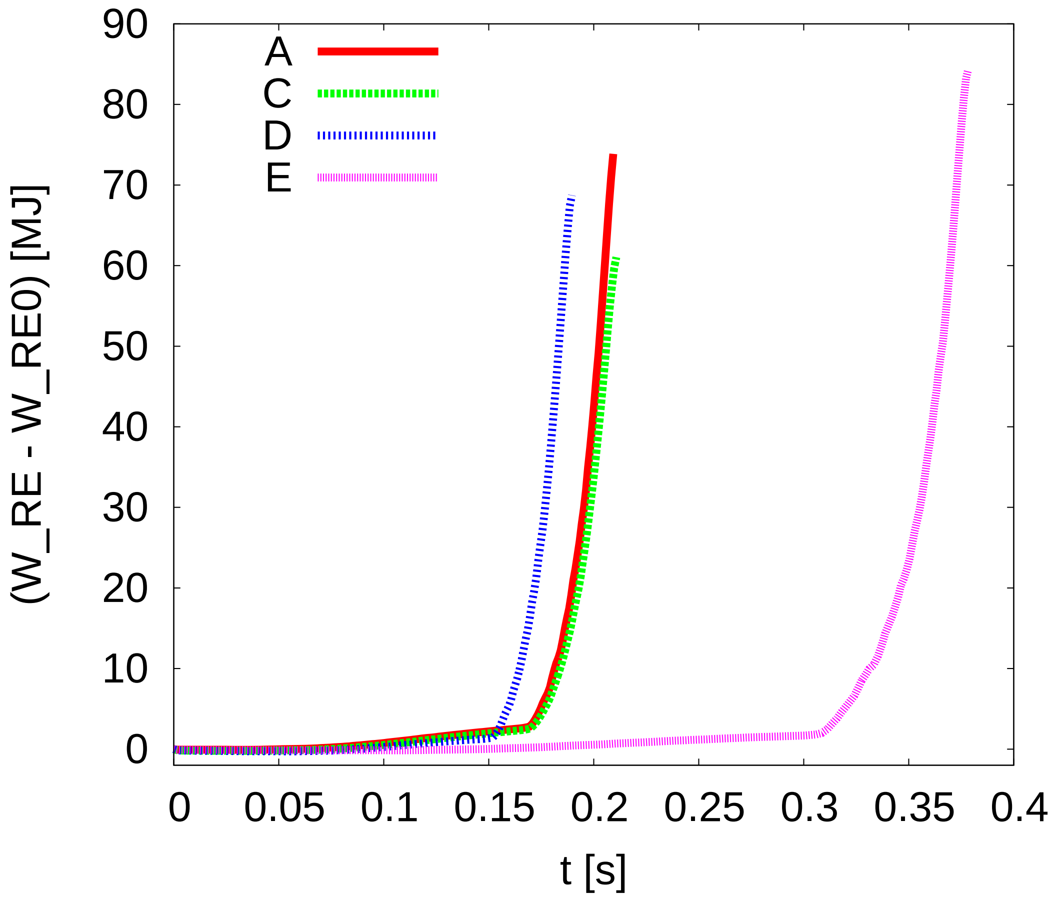
E $\sigma_{\mathrm{wall}}^{\mathrm{E}} = 2\,\sigma_{\mathrm{wall}}^{\mathrm{A}}$

$\Rightarrow$ τ_{wall} larger

$\Rightarrow$ longer duration

small differences in ΔW_{RE} found!

parameter dependence of RE energy gain



Numerical Results - Scrape-off Loss Power

A reference case

$I_\mathrm{P}^0 = 10\,\mathrm{MA}$ (flat profile)

$T \simeq 5\,\mathrm{eV}$

$n \simeq 10^{21}\,\mathrm{m}^{-3}$

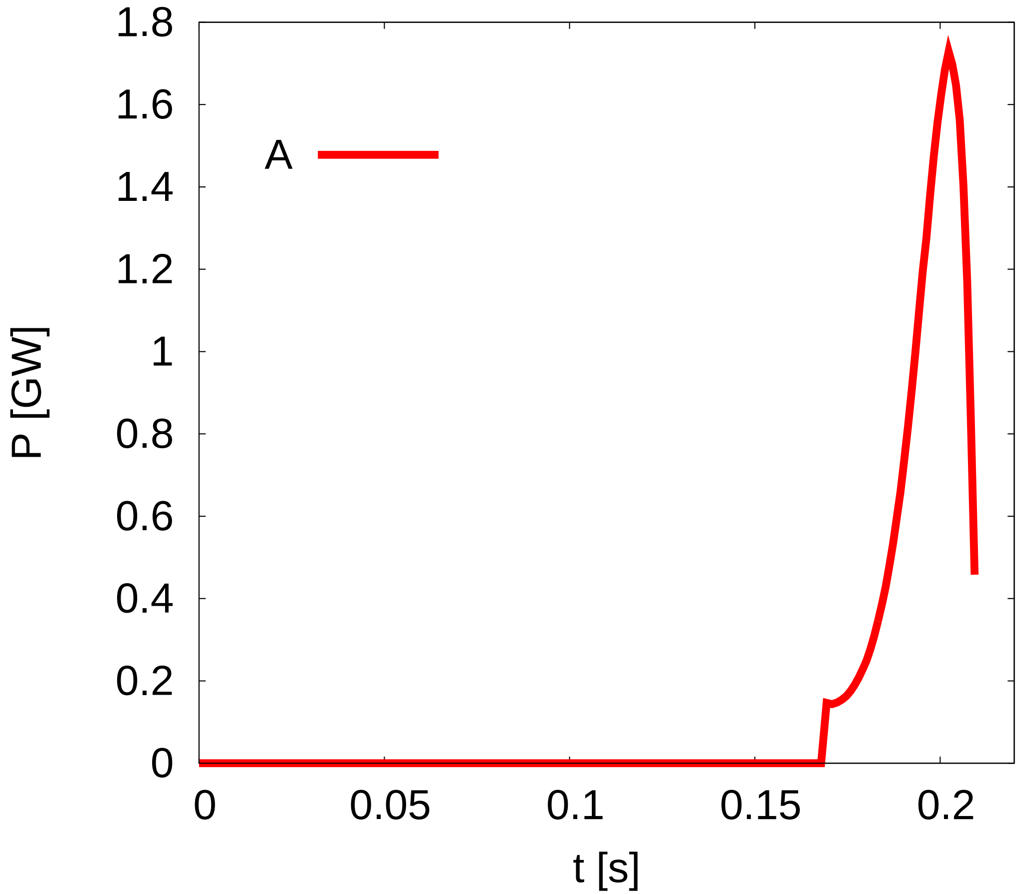
$I_\mathrm{PF1} = 0.84\,I_\mathrm{P}^0$

$$P \approx (2\pi)^2 R_0\, \varrho\, n_\mathrm{r}^\mathrm{edge}\, v_z\, (mc^2 \ln \Lambda)$$

- onset with scrape-off phase
- strong peak (... can be worse!)
- size of wall fragment?
- severe damage to walls!

no material can withstand $\mathrm{GW/m}^2$

scrape-off loss power $P(t)$



Summary

- 2D model for energy conversion under disruption presented
- earlier qualitative results by other authors confirmed
- substantial conversion of magnetic energy during disruptions
- final RE energies of up to $\sim 100\,\mathrm{MJ}$ possible
- two qualitatively different phases of plasma motion found
- energy mainly consumed by friction in free-motion phase
- strong energy gain by REs during scrape-off phase

RE suppression/control/mitigation is a key topic for ITER!

(submitted to *Physics of Plasmas*)